EEL5701 DSP – Summer 98
Final Exam   August 6, 1998

NAME: ________________________________

This exam is open-book and calculator. You may use any books or papers that you like. There are three problems in this exam, you have have three full hours for this exam though you should only need two. State your assumptions and reasoning for each problem. Justify your steps and clearly indicate your final answers.

Finally, I hope you had a good semester. Check the course website for the final grade distributions.

1	
2	
3	
TOTAL	

1. (35 points) This problem studies the design of an FIR approximation to an all-pass filter. You are given the following stable, causal, all-pass, IIR filter:

$$H_d(z) = \frac{1 - 2z^{-1}}{1 - .5z^{-1}}$$

As you know, the frequency response has a constant magnitude for all frequencies. You should easily be able to calculate the magnitude response in this case as $|H_d(e^{j\omega})| = 2$

(a) (10 points) Derive the impulse response for $H_d(z)$. (Hint: if you can't derive the general impulse response, at least calculate a few terms using long division). Make sure you spend the time to get this problem right since other questions rely on this result.

use this space if you need to show more work

(b) (10 points) A rectangular window is used to retain exactly L terms of the impulse response. Derive an expression for the $H(z)$ of the truncated filter. Make sure that you simplify $H(z)$ as much as you can, i.e. no long summations.

(c) (15 points) You are told that the maximum deviation from the constant magnitude response occurs at $\omega = 0$. What is the minimum value of L (number of terms) such that the magnitude response does not deviate by more than 1% of its designed value (i.e. 2)?

2. (35 points) This problem studies the design of an FIR all-pass filter using the frequency sampling approach. Remember that in this approach, you uniformly sample the frequency response at L equally spaced points and then take the inverse DFT to obtain $h[n]$. You are given the same filter as in problem 1:

$$H_d(z) = \frac{1 - 2z^{-1}}{1 - .5z^{-1}}$$

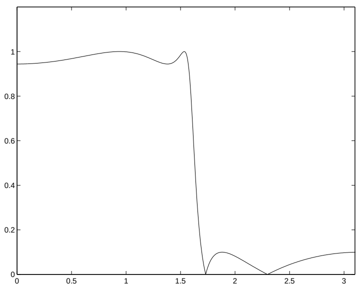
(a) (10 points) Derive $H[k]$, the L-point DFT of your L-point impulse response $h[n]$

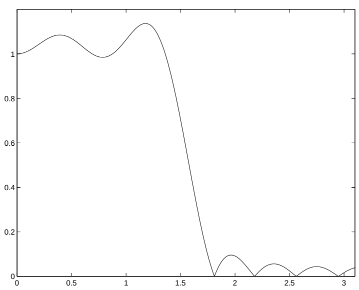
(b) (15 points) Derive $h[n]$, the L-point impulse response. Simplify your result as much as possible.

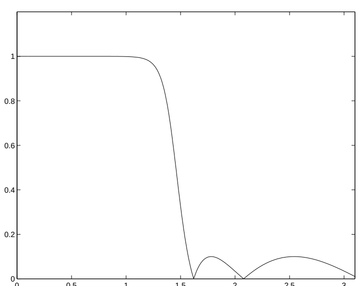
(c) (10 points) What is the magnitude of the frequency response of your new filter $H(e^{j\omega})$ at $\omega = 0$? How do you think the overall performance of this filter will compare to filters of the same order using a rectangular window (like in problem 1) and using Parks-McClellan?

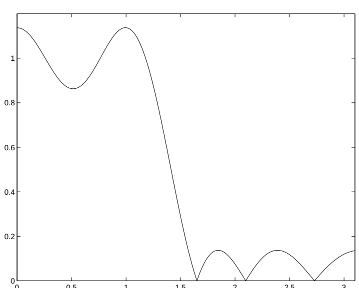
3. (30 points) Short Answer

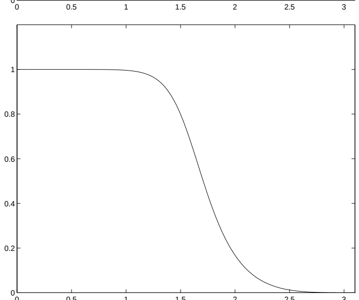
(a) (5 points) Five filters were designed. The FIR filters used the rectangular window method and Parks-McClellan. The IIR filters used the bilinear transform from the continuous-time Butterworth, Chebyshev, and Elliptical filters. Label each plot with the correct method. (Each plot shows the magnitude of the response vs. ω.)











(b) (5 points) Suppose you are given a low-pass, discrete-time filter $H(z)$. What type of filter (low-pass, band-pass, etc.) will the following spectral transformation produce?

$$z = 2\frac{\hat{z} - .5}{\hat{z} - 2}$$

(c) (5 points) Can a type I linear-phase filter implement a high-pass filter? Explain why or why not.

(d) (5 points) A sound lasts for exactly 1 second. The highest frequency in the sound occurs at 20kHz. What is the smallest frequency resolution (Δf) that can be achieved via sampling and taking a DFT of the resulting samples?

(e) (5 points) Derive the impulse response of an FIR filter with a frequency response that is never zero but has minimum amplitude at $\omega = \pi/4$.

(f) (5 points) Draw a 4-point radix-2 decimation-in-time FFT signal flow diagram. Explicitly label the inputs, the outputs, and the multiplying factors. Show the computation of the FFT of [1,1,1,1] by explicitly labeling the signals at each node. (Make sure that the $X[k]$ you end up computing is correct!)