level of performance is achieved only for $\sigma_e < 32$. It should be pointed out that the numerical results here presented were obtained without the use of error correcting codes.

\[ E_b/N_0 = 4 \text{dB} \]
\[ E_b/N_0 = 6 \text{dB} \]
\[ E_b/N_0 = 10 \text{dB} \]
\[ E_b/N_0 = 12 \text{dB} \]

Fig. 1 Error probabilities for joint detection receiver against $\sigma_e$ for two users sharing timeslot

Fig. 2 Error probabilities for joint detection receiver against $\sigma_e$ for eight users sharing timeslot

F&H filter: A novel ultra-low power discrete time filter

V.M. Grade Tavares, J.C. Principe and J.G. Harris

A novel technique for designing filters with long time constants in the discrete time domain is presented. The F&H (filter and hold) methodology halts the state of a continuous time filter every $T$ seconds resulting in a filter implementation with time constants that can be controlled in three distinct ways: by the sampling period $T$, the duty cycle $\kappa = \tau/T$ or the time constant of the continuous time filter prototype. The final filter can be constructed from a typical Gm-C technique with very low power consumption.

Introduction: Among the different design methodologies of filters for sampled signals, the SC (switched capacitor) [1] and SI (switched current) [2] filters are the most commonly utilised. SC designs are sampled time filters with very high precision time constants. They rely on capacitor ratios which can be made very precise [3]. SI techniques have evolved to cope with technological trends dictated by digital VLSI technology (such as low power voltages), which is prevalent today. SI techniques do not rely on linear capacitors to perform calculations and are suitable for standard technologies.

Nevertheless both SC and SI circuits still consume large amounts of power. Furthermore if large time constants are required for the filters, high ratios of capacitors or transistors are needed for these conventional techniques.

In this Letter we propose a third design methodology for filtering. It consists of a mixed analogue-discrete time design that combines analogue continuous time conventional filtering with sampling. Long time constants are obtained with relatively low area due to parameter scaling. Simulated results are shown for a second order cascaded filter with bias currents of the order of nanosamps.

F&H filter: The F&H design relies on sampling the states of a continuous time filter as follows. Consider a one pole system as in Fig. 1. Switch $S_n$ is controlled by the clock $\Phi$ with period $T$ and closure duration $\tau$ creating a duty cycle of $k = \kappa T$. Assume that $V_o(nT)$ is constant in the interval $t \in nT - \Delta < t \leq nT + \Delta + \Delta$. The time constant $\tau$ is a vanishingly small number. Then $V_o(nT + \tau)$ is the step response of the RC network after $T$ seconds given by eqn. 1:

$V_o(nT + \tau) = V_o(nT) + [V_o(nT) - V_o(nT + \tau)] e^{-\frac{\tau}{RC}}$

$V_o(nT + \tau) = V_o[nT] - V_o[nT + \tau] e^{-\frac{\tau}{RC}}$

$V_o(nT + \tau) = V_o[nT + \tau] e^{-\frac{\tau}{RC}}$

At the end of the sampling interval $T$, the switch opens at $\tau = nT + \Delta$ and so that voltage is held in the capacitor until the next update. The difference equation (eqn. 1) yields a transfer function $H(z)$ defined by

$H(z) = \frac{1 - e^{-\frac{\tau}{RC} z^{-1}}}{1 - e^{-\frac{\tau}{RC} z^{-1}}}$

Since $\kappa < 1$ the time constant can be made slower by the factor $\kappa^2$, effectively implementing a much slower cutoff lowpass filter.

Second order F&H filter example: Consider now the second-order differential equation defined by eqn. 2 with time constants $a = 2200\pi$ (350Hz) and $b = 7200\pi$ (1146Hz). Eqn. 3 is part of a non-linear dynamical system that models the mammalian olfactory system [4] ($a'$ and $b'$ are real):

$1 = a \cdot b \left( \frac{\partial^2 x}{\partial t^2} + (a + b) \frac{\partial x}{\partial t} + a \cdot b \cdot x \right) = f(t)$

References

This differential equation can be approximated by a cascade of two lowpass discrete time filters such as the F&H circuit shown in Fig. 2. The op-amp symbol is simply a transconductance amplifier (simple differential pair just as in a Gm-C filter [5]). The constants \( a \) and \( b \) are defined by

\[
a = a_1 \cdot k = \frac{Gm1}{C1} \cdot k \quad b = b_1 \cdot k = \frac{Gm2}{C2} \cdot k
\]

For the present simulations MOSIS 1.2um technology was used. The duty cycle was made to be \( k = 0.1 \) which implies the use of time constants of \( a_1 = 22k\Omega (3.5kHz) \) and \( b_1 = 72k\Omega (11.46kHz) \). The OTAs have a fixed 10nA bias current with capacitors \( C1 = 1pF \) and \( C2 = 3.3pF \). Fig. 3 shows the simulation result for the F&H second order filter and the corresponding continuous time filter with capacitors 10 times bigger than those used for the F&H filter.

\[\text{Fig. 2 Second-order F&H lowpass filter}\]

\[\text{Fig. 3 F&H against continuous time Gm-C filter result for } k = 0.1\]

(i) continuous time Gm-C filter output (capacitors ten times bigger)
(ii) input
(iii) F&H output (staircase shape)

\[\text{Fig. 4 F&H against continuous time Gm-C filter result for } k = 0.01\]

(i) continuous time Gm-C filter output (capacitors one hundred times bigger)
(ii) input
(iii) F&H output (staircase shape)

**Lowering the duty cycle:** The same filter as in Fig. 2 is used to implement the filter defined in eqn. 3 but now with \( a = 220\Omega (35Hz) \) and \( b = 720\Omega (114.6Hz) \). Only the duty cycle is changed to \( k = 0.01 \). The time constants are then scaled up by a factor of 100. At this low duty cycle the clock-feedthrough acts as an injected error to the time constant. Trimming the time constants with the duty cycle compensates for this effect to a large extent. Fig. 4 shows the result after trimming \( k \) (final \( k = 0.0093 \)).

**Conclusion:** A new type of discrete time lowpass filter has been presented. The power consumption of this type of filter can be made extremely low; for example, in the second order filter example shown, the power consumption is ~100nW. There are three distinct ways of fine tuning the filter: by the duty cycle \( k \), the time period \( T \) and by \( Gm \), or a combination of the three. The time constants are scalable by the duty cycle, which allows low frequency filters to be built with easily achievable time constants for the target technology with no change in the sampling period.

Lowpass filters with large time constants are required for biological systems modelling [6]. For those models to be implemented efficiently new simple techniques of filtering are needed, such as the technique suggested in this Letter. This is also the case in other important applications such as adaptive processing based on dispersive delay lines such as the Gamma filter [7].

Although not addressed in this Letter, the F&H technique is general (HP, LP, BP, BR) and can be applied to a wide range of different active filter topologies of any order. It also works well for non-sampled input signals provided that the output is passed through a smoothing filter (at \( \pi/T \)) and the input is bandlimited to \( \pi/T \). The resulting filter is equivalent to a continuous time filter with time constants 1/10 times longer.

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**References**


**Outage performance of cellular systems over arbitrary lognormal shadowed Rician channels**

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Techniques for analysing the outage performance for lognormal shadowed Rician fading channels are becoming of increasing importance as the concept of micro-cells is introduced into wireless personal communications. A general solution, however, has not yet been developed except for some special cases. The authors present an exact solution which takes the form of two-fold integrals, and is applicable to general situations without any restrictions.