Design and Implementation of Linear Phase FIR Filters for Biological Signal Processing

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Abstract—This paper presents a new method for designing digital linear phase, finite impulse response filters with loose frequency response characteristics, but with good time resolution as is required in biological signal conditioning. The design is very simple and has been used with success in the microcomputer implementation of filters for the automated processing of electroencephalographic (EEG) data. Examples and a discussion of possible filter implementations are included.

I. INTRODUCTION

BIOLOGICAL signals display an extremely large variability and complexity but their spectra are limited to a few kilohertz (in fact, most are limited way below the kilohertz mark). The relatively low maximum frequency implies that the required sampling frequency for real-time data analysis is within the reach of most microprocessor based systems provided the signal processing methodology is kept simple (in real-time applications the processing of one sample must be completed before the next sample is acquired).

With the widespread use of microprocessors in the laboratory, it is interesting to study filter designs and microcomputer implementation methodologies which will be easily understood by signal processing nonexperts. The considerations that follow address the filtering characteristics and the implementation constraints for the processing of a class of biological signals using microcomputers, which led to the development of a new filter design method called the stopband.

A. Characteristics of EEG Filters

The real-time processing of electroencephalographic data in sleep studies involves the detection of phasic events (delta, theta, alpha, sigma, and beta waves), which can be considered transient waveforms. Time domain parameterization of these waveforms has been a very productive method of EEG signal detection and quantification during sleep [1]. The block diagram of the signal processing scheme is presented in Fig. 1.

The input signal is conditioned by means of linear filters, with two main purposes:

- increase the repeatability of the measurements through the attenuation of out-of-band components (high frequency noise and low frequency artifacts), thus creating robust performance;
- ease the implementation of the feature extraction algorithms. An example is to use simple zero crossing detection to measure waveform frequency (defined as the inverse of the period).

The attenuation of out-of-band components must be met with filters displaying good time response, i.e., linear phase and short impulse response in order to disturb the least the time domain features of the in-band waves measured from the filtered data. The typical filtering problem is formulated in a different way since one requires filters with nearly flat passbands, sharp transition bandwidth, and large out-of-band attenuation, i.e., only frequency domain restrictions; the impulse response of such filters displays large oscillations which die down slowly. The conclusion is that this type of filter is unacceptable for the signal conditions referred to above. However, it is still an open problem to formulate precisely an optimality criteria for filters used in signal conditioning. We resorted to experimental evaluation of a good candidate, the class of broad-band finite impulse response (FIR), linear-phase filters. Preliminary results showed that they work well for the kind of EEG waveform processing described in Fig. 1 [2].

B. Filter Design for Microcomputer Implementation

The digital filter literature describes basically three types of FIR design procedures: the Fourier series method (also called the window method), numerical optimization methods, and the frequency sampling method. In the Fourier series method one takes the inverse Fourier transform of the required frequency response $H(e^{j\omega T})$ of the filter and obtains after truncation and appropriate shifting a causal impulse response $h(nT)$. The filter coefficients are, therefore, the Fourier series expansion (in time) of the periodic frequency response of the digital network. The frequency characteristics of the digital filter (filter types, center frequency, bandwidth, out-of-band attenuation) are embedded in the frequency response utilized and also in the number of terms kept in the series expansion [3].

The second technique, numerical optimization methods, searches for zero placements that yield desirable at-
tenation characteristics. The Remez algorithm is often utilized [4] but other interesting approaches, including integer coefficient approximation, have been presented [5]. The frequency sampling method creates a filter in two steps [6]. First the filter stopband level is created by placing a sufficient number of zeros around the unit circle of the z-plane. Some of the zeros are subsequently cancelled with poles (also on the unit circle) to create the prescribed filter type. The number of zeros is also related to the filter transition bandwidth. This implementation is efficient for narrow-band low-pass, high, or bandpass filter types. It also has the advantage of greater simplicity. Its drawback is a potential instability if imperfect pole zero cancellation occurs [7], which is highly probable for small wordlength implementations.

To achieve the sampling frequency necessary for biological signal processing using microcomputers, two factors must be addressed: the type of arithmetic and the complexity of the filter structures. It turns out that the filter computation must be performed in fixed point arithmetic to utilize efficiently the primitive microprocessor ALU. With this choice, algorithm complexity is dominated by the number of multiplications because they take much longer to execute than additions and delays (i.e., memory transfers). Also, only low-order filter structures may be considered due to the relative slow operation speed.

The Remez algorithm designs optimal filters in terms of frequency domain specifications alone which does not suit signal conditioning. Also, in general, optimal methods are very sensitive to finite length effects so degradation of frequency response occurs for fixed point implementations. Even larger degradation can be expected if the multiplications are approximated by a small number of shift-and-add operations.

A design alternative is to accept certain limitations in the frequency response characteristics but develop filter design procedures which can be efficiently implemented in microcomputer systems and easily understood by their users. A good example is the frequency sampling method already described, but due to the intrinsic variability of biological signals, narrow-band filters do not seem to be the most appropriate. We propose a new design method that can be thought as the dual of frequency sampling. The filter transfer function is also constructed in two steps. First, zeros are placed in the unit circle creating several passbands, one of which corresponds to the desired passband; then, zeros are placed in the unit circle in order to squelch the gain at the unwanted passbands. Since poles are not used, filter instability will never occur. Memory and one addition are used to implement roughly half of the zeros, which means fast operation. Also, there is some flexibility to move slightly the stopband zeros (at a cost of decreasing the stopband attenuation) in order to save multiplications. This procedure is very well known and consists in approximating the multiplications by a small number of shift-and-adds. The resulting imprecision can be afforded in the stopband method because it only affects the stopband attenuation, but it is not recommended in optimal filter design or in the frequency sampling method because improper cancellation of zeros with poles in the unit circle give rise to oscillations.

The design outlined above can be accomplished with few equations requiring only pencil and paper to arrive at the filter transfer function.

II. A Class of Bandpass Filters

The bandpass filters described in this paper can be introduced by considering the filter recursion relation

\[ y_n = x_n + x_{n-N} \]  

(1)

where \( y_n \) represents the current filter output value, \( x_n \) represents the current filter input, and \( x_{n-N} \) the filter input \( N \) samples earlier. This time-domain description of the filter can be written in the frequency domain as

\[ Y(z) = (1 + z^{-N}) X(z), \]  

(2)

from which the filter transfer function \( G(z) \) is defined as

\[ G(z) = \frac{Y(z)}{X(z)} = (1 + z^{-N}). \]  

(3)

This is a finite impulse response (FIR) filter with all of its zeros uniformly spaced around the unit circle (\( |z| = 1 \)).
The absolute value of the frequency response is

\[ |G(z)|_{z=e^{j\omega T}} = |1 + e^{-jN\omega T}| = 2 \left| e^{-jN\omega T} \right| \cos \left( \frac{N\omega T}{2} \right) \]

\[ = 2 \left| e^{-jN\omega T} \right| \cos \left( \frac{N\omega T}{2} \right) \]  

(4)

i.e., the modulus of a cosine function. The filter phase shift is

\[ \arg G(e^{j\omega T}) = -\frac{N\omega T}{2} \]  

(5)

which is a linear function of frequency. The frequency response of this filter (obtained by setting \( z = e^{j\omega T} \)) is shown in Fig. 2. The filter phase shift is linear and the magnitude of the frequency response appears as a rectified sinewave consisting of \( N \) arcades between \( \omega = 0 \) and \( \omega = \omega_1 \). It can be interpreted as a bandpass filter with multiple bandpasses. The number of peaks plus zeros of the response function [see (4)] between 0 and \( \omega/2 \) is \( N + 1 \) with the peaks appearing equidistant between the zeros. For \( N \) odd, the number of peaks and zeros are equal. A filter with a single bandpass can be achieved cancelling all but one of the arcades by adding additional zeros to the transfer function. Fig. 3 illustrates the procedure. Fig. 3(a) illustrates the magnitude of the frequency response of \( G(z) \) for \( N = 3 \). If a zero is added at the origin of the frequency axis (\( z = 1 \)) [Fig. 3(b)], this attenuates the low frequency response of the filter resulting in a bandpass filter when one cascades both sections. The complete filter transfer function will be

\[ H(z) = (1 - z^{-1}) \ G(z) = (1 - z^{-1})(1 + z^{-3}). \]

and its frequency response, illustrated in Fig. 3(c), is a bandpass characteristic with a center frequency \( \omega_1/3 \). (Note that in digital filters with real coefficients, \( |H(\omega)| \) is periodic with \( \omega_1 \) and an even function, so it is sufficient to describe the frequency response up to the frequency \( \omega_1/2 \).)

The bandpass filter design procedure consists of selecting \( N \), the sampling frequency, and the number and location of the additional zeros to shape the frequency response. \( G(z) \) is referred to as the primary resonator in the following discussion.

III. BANDPASS FILTER DESIGN

The peak frequencies of (4) are

\[ \omega_p = \frac{K\omega_1}{N} \quad K = 0, 1, 2, \ldots \]  

(6)

The bandwidth of one arcade of (4) is

\[ B = \frac{\omega_1}{2N} \]  

(7)

since we are dealing with the magnitude of a sinusoidal function and \( \sin(\pi/4) = 0.707 \).

How are these facts used to design bandpass filters? If one of the nonzero frequency peaks of (4) is selected as the filter center frequency, the sampling frequency can be expressed in terms of the filter center frequency as

\[ \omega_s = \frac{N\omega_p}{K} \quad K = 1, 2, \ldots, \text{INT} \left( \frac{N - 1}{2} \right) \]  

(8)

where \( \text{INT} \ (N - 1)/2 \) means the largest integer contained in \( (N - 1)/2 \).

The first step of the filter design is to select \( N \) and the sampling frequency \( \omega_s \). \( N \) should be as small as possible.
for filter simplicity; the minimum sampling frequency is determined by the frequency of the analog signal and the information to be extracted from the signal.

Choose \( K = K_0 \) such that one of the peaks coincides with the prescribed filter center frequency \( \omega_c \). That is

\[
K_0 = \frac{N\omega_c}{\omega_s}. \tag{9}
\]

The integer \( N \) is then determined from (7)

\[
N = \frac{\omega_s}{2B}. \tag{10}
\]

The narrower the desired bandwidth, the larger must be the filter order.

It is seen that the filter center frequency must be an integer multiple of the filter bandwidth. If this is not the case, it is necessary to offset slightly \( \omega_p \) or \( B \) (or both) such that the above condition holds. Also, only symmetric bandpass filters with a center frequency greater than the bandwidth can be realized.

Up to this point we have described how to choose \( N \) and \( K_0 \) to create a passband centered at a given frequency and with a prescribed bandwidth. However, this results in multiple passbands. For certain applications some of the other passbands may not be of concern [6], i.e., the input signal may have negligible energy where the passbands occur, but generally this is not the case. To create a single passband filter, additional zeros must be added to attenuate the unwanted passbands. The linear phase characteristic will be preserved if zeros described by the function

\[
H_i(z) = z^{-2} - 2 \cos \phi_i z^{-1} + 1 \tag{11}
\]

are generated. This equation represents a pair of complex conjugate zeros on the unit circle of the \( z \) plane. The \( \phi_i \) are chosen such that the zeros' locations coincide with the maxima of (4) which one wants to suppress, i.e.,

\[
\phi_i = \frac{2K}{N} \pi \left\{ K = 1, 2, \cdots, \text{INT} \left( \frac{N - 1}{2} \right) \right\} K \neq K_0. \tag{12}
\]

For a bandpass filter generated with the primary resonator described by (3) the total filter order is \( 2N - 2 \). The additional resonators must include the first-order function \( H_1(z) = z^{-1} - 1 \) which creates a zero at \( z = 1 \). For \( N \) even a resonator must also be included at \( z = -1 \) and the two first-order functions can be written as

\[
H_i(z) = z^{-2} - 1.
\]

So, for example, if \( N = 4 \), the total filter order will be 6, the primary resonator is fourth-order and two more zeros are added \( (z = \pm 1) \). The additional resonators alter in a nonsymmetrical way the filter passband, but the effect is slight, since the transfer function for each resonator is

\[
|H_i(e^{j\omega T})| = 2|\cos \omega T - \cos \phi_i|. \tag{13}
\]

\[\text{Fig. 4. Frequency response of the linear phase filter } G(z) = 1 - z^{-N}.\]

Another Class of Bandpass Filters

Another equation which realizes \( N \) zeros equally spaced on the unit circle with \( N \) delays and one addition (subtraction) is

\[
G(z) = 1 - z^{-N} \tag{14}
\]

which has the absolute value

\[
|1 - z^{-N}|_{z = e^{j\omega T}} = |1 - e^{-jN\omega T}| = 2 |\sin \frac{N\omega T}{2}| \left| e^{-jN\omega T/2} \right| \tag{15}
\]

i.e., the module of a sinusoidal function. The phase shift of this function,

\[
\arg G(e^{j\omega T}) = \frac{-N\omega T}{2} + \frac{\pi}{2} \tag{16}
\]

is also a linear function of frequency. The frequency response described by (15) is illustrated in Fig. 4. The maxima of this magnitude response occur at

\[
\omega_p = \frac{(2K + 1)\omega_s}{2N}, \quad K = 0, 1, \cdots. \tag{17}
\]

The primary resonator has the advantage over the resonator described by (3) that its frequency response is zero at \( \omega = 0 \), so a zero does not need to be added at \( z = 1 \). The bandwidth for this response is also given by (7). For this function the sampling frequency, in terms of the filter center frequency, is

\[
\omega_s = \frac{2N\omega_c}{2K + 1}, \quad K = 0, 1, \cdots. \text{INT} \left( \frac{N}{2} \right) - 1. \tag{18}
\]

To choose a maxima as a filter center frequency, make \( K = K_0 \) with

\[
K_0 = \frac{N\omega_c}{\omega_s} = \frac{1}{2}. \tag{19}
\]

Equation (11) can also be used here to attenuate the
other passbands with
\[
\phi_i = \frac{2K + 1}{N} \pi \left\{ \begin{array}{ll}
K = 0, 1, \cdots, \text{INT} \left( \frac{N}{2} \right) - 1 \\
K \neq K_0.
\end{array} \right.
\]
\[ (20) \]
For \( N \) odd an extra resonator described by the equation \( H(z) = z^{-1} + 1 \) must be added.

The function described by (14) in fact allows the designer to choose filters with the possible center frequencies shifted by \( \pi/N \) from the ones given by (3). Therefore, there are \( (N - 1) \) possibilities for placing the center frequency for a primary resonator of order \( N \), which minimizes the disadvantage of the coupling between \( B \) and \( \omega_c \) inherent in the design method.

**Filter Structure**

The filter realization in a cascade form is straightforward. Without loss of generality we assume only simple zeros are added. The complete filter transfer function \( H(z) \)
\[ H(z) = G(z) \prod_i H_i(z) \]  
(21)
where the primary resonator \( G(z) \) is determined by (3) or (14) and the \( H_i(z) \) are described by (11).

The block diagram realization of (21) is shown in Fig. 5 (with the initial delays omitted). The cascade implementation of an \( (2N - 2) \)-th-order filter of this type requires at most \( (N - 2) \)/2 multipliers, \( N - 1 \) adders, and \( 2N - 2 \) delays.

Faster operation is possible if the filters do not require multiplications. This will be the case if \( 2 \cos \phi_i \) is approximated by
\[
2 \cos \phi_i \approx r_1 + r_2 2^{-1} + r_3 2^{-1} \quad (r_i = 0, 1, \text{or } -1)
\]
(23)
since the resonator coefficient can then be implemented with shift and add operations which are much faster than a multiplication.

Table I contains a 3 bit approximation (at most two shifts and two addition operations) to \( 2 \cos \phi_i \) with an error < 6.5° for every possible zero location of (3) or (14) for \( N \) up to 8. A 4 bit approximation gives substantially more precision (or keeps the precision in the zero location for higher order functions), but this approximation is not considered here since it requires three shifts and three additions and starts approximating the computation time of a multiplication. The filters described here are relatively insensitive to the bandstop zero locations and the approximations given in Table I are usually sufficient.

To summarize the design procedure:

1) Select the filter center frequency \( \omega_c \) to be an integer multiple of the bandwidth
\[
\omega_c = 2B \left( \frac{K + \alpha}{2} \right)
\]
(24)
where the parameter \( \alpha \) is 0 or 1. If \( \alpha = 0 \) the primary resonator is described by (3), for \( \alpha = 1 \) (14) describes the primary resonator. The minimum sampling rate is determined by other factors in the signal processing problem.

2) Select the sampling frequency \( \omega_s \) and \( N \) so that
\[
\omega_p = \frac{\omega_s}{N} \left( \frac{K + \alpha}{2} \right).
\]
(25)
This step also fixes the passband (one of the arcs of the sinusoidal function).

3) A single passband function is then realized by generating resonators given by (11), with \( \phi_i \) calculated by (12) or (20). For each pair of unwanted lobes one resonator must be implemented. To avoid a multiplication in each resonator the values in Table I should be used.

4) The final filter of order \( 2N - 2 \) will be a cascade of the filter described by (3) or (14) with the additional resonators described by (11).

The shift in the filter's center frequency produced by cascading second-order structures [see (13)] can be calculated. The equation for the maxima of the composite function, when the principal resonator is given by (4), is given by
\[
\tan \frac{N \omega_T}{2} = \frac{2}{N \cos \phi_i - \cos \omega_T}
\]
and, when the principal resonator is defined by (15), is
\[
\cot \frac{N \omega_T}{2} = \frac{2}{N \cos \omega_T - \cos \phi_i}.
\]
Therefore, the resonator pushes the frequency of the maxima away from the resonator zero. The effect depends upon the primary resonator order and the resonator zero placement, being the same for both equations.

In terms of the distance between the resonator zero and the maxima, and for primary resonator orders between 4
Table II

<table>
<thead>
<tr>
<th>primary resonator order</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>nearest resonator (percent)</td>
<td>6.5</td>
<td>5.7</td>
<td>5.3</td>
<td>5.0</td>
<td>4.9</td>
</tr>
<tr>
<td>second nearest (percent)</td>
<td>1.4</td>
<td>1.2</td>
<td>1.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>third nearest (percent)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>

(a)

Fig. 6. (a) A sixth-order bandpass filter. (b) Frequency and impulse response of the filter illustrated in Fig. 6(a).

and 8, the percentage shift in frequency is depicted in Table II. The overall effect when more than one additional resonator is used can be estimated (maximized) by algebraically adding the shifts.

Example: Design a filter with a center frequency $f_c = 14$ Hz and a 14 Hz bandwidth. The minimum sampling frequency is 100 Hz. Solution: We see from (24) that $K = 0$ and $\alpha = 1$, since $f_c = B$. The smallest $N$ such that $f_s > 100$ Hz is $N = 4$ and $f_s = 112$ Hz is obtained using (14) for the primary resonator. That is

$$G(z) = 1 - z^{-4}.$$  

This function has two pairs of lobes around the unit circle, the lowest frequency pair ($K = 0$) is the passband and the other pair suppressed. One resonator is needed to suppress the second lobe. From (20) with $K = 1$

$$\phi_i = \frac{3\pi}{4} = 135^\circ$$

which can be approximated by $138^\circ$ (Table I), giving a coefficient of $2 \cos \phi_i = -1.5$.

The equation to implement the bandpass filter is

$$H(z) = (z^{-4} - 1)(z^{-2} + 1.5z^{-1} + 1)$$

which is realized as in Fig. 6(a) with no multipliers. Fig. 6(b) shows a computer simulation of the frequency response along with the impulse response and the zero location in the $z$ plane [9].

To demonstrate the flexibility of the design procedure, suppose that the frequency response of Fig. 6(b) is not
acceptable because it does not have sufficient attenuation in the stopband. The attenuation can be increased by placing additional zeros midway between the existing stopband zeros, i.e., zeros at 90° + 22.5° = 112.5° and 157.5°. This can be accomplished (using the approximations provided by Table 1) with a cascade of two additional resonators with equations
\[ z^{-2} + 0.75z^{-1} + 1 \]
and
\[ z^{-2} + 1.75z^{-1} + 1. \]

The final transfer function is
\[ H(z) = (z^{-4} - 1)(z^{-2} + 1.5z^{-1} + 1) \cdot (z^{-2} + 0.75z^{-1} + 1)(z^{-2} + 1.75z^{-1} + 1). \]

The frequency response is illustrated in Fig. 7.

**Visualization of the Design in the Z Plane**

Notice that (6), (7), and (17) reduce the analog filter frequency parameters to angles in the z plane, if \( \omega_z \) is thought of as \( 2\pi \) or 360°. The zero placement of (3) and (14) can be easily found: divide the unit circle in \( N \) equal sectors \( (2\pi/N) \) (Fig. 4); for (14) an additional rotation of \( \pi/N \) is required, as Fig. 2 shows. Center frequencies occur midway between the zeros. Choose which equation, (3) or (14), applies (step 1) and determine \( N \) (step 2). Place the filter center frequency in the appropriate sector (for the example \( B = f_c = 360/8 = 45^\circ \)). To eliminate the unwanted passbands, generate additional resonators by placing zeros midway between the primary resonator zeros. For this example, one pair of zeros at \( \pm(90^\circ + (180 - 90)/2) = \pm135^\circ \) must be generated (Fig. 8). Read from Table 1 the closest \( \phi_i \) (138°) and use the indicated value for 2 cos \( \phi_i \) to save multiplications. This procedure can be iterated if higher out-of-band attenuation is required (just place zeros midway of the existing stopband zeros).

**Cascade the principal resonator with the additional resonators.**

**IV. DESIGN OF LOW-PASS FILTERS/HIGH-PASS FILTERS**

To design a low-pass filter it is required to have a passband centered at zero frequency; therefore, only (3) can be utilized for the primary resonator. In the low-pass filter the design parameter is the bandwidth \( B \) which is
\[ B = \frac{\omega_z}{4N}. \]  

The \( N - 1 \) higher frequency lobes are then removed in exactly the same manner as in the bandpass case, i.e., add resonators described by (11) with the \( \phi_i \) given by (12). The complete filter will be of order \( 2N - 1 \).

The high-pass case is similar to the low-pass except that now we require a peak of (3) or (14) at one-half of the sampling frequency (\( \omega_s/2 \)). For the digital high-pass filter the intrinsic periodicity of the frequency response of a digital system results in a magnitude as illustrated in Fig. 8.
9. From the figure, the high-pass filter \(-3\) dB frequency is

\[ \omega_L = \frac{\omega_s}{2} - B \]  \(\text{(27)}\)

where \(B\) is given by (26). The order of the primary resonator is chosen in the same way as step two of the bandpass design. If \(N\) is even, the primary resonator described by (3) must be used. In \(N\) is odd, then (14) must be used. Resonators are determined by (12) or (20), respectively, and are still required to remove the low frequency lobes.

V. PROCEDURE TO DESIGN BAND REJECT FILTERS

Band reject filters are not easily designed with this method. Nevertheless, one can create a bandstop function from a bandpass by calculating

\[ H(e^{j\omega T})_{\text{max}} - H(\text{bandpass}). \]

Therefore, the bandstop filter design reduces to the design of a bandpass filter. This method can also be used to design high-pass filters from low-pass ones. However, the important linear phase characteristic is lost.

VI. IMPLEMENTATION OF FIR FILTERS IN A MICROCOMPUTER

Up to here we have presented block diagrams of filter structures. We will take the example of Fig. 6(a) which realizes the bandpass function of the example to illustrate its implementation in a microcomputer.

In this case we have a sixth-order filter (the order of the polynomial in \(z^{-1}\)). Analyzing Fig. 6(a) we conclude that the output \(y_n\) is computed using the input \(x_n\) and intermediate results (eventually past values of the input). An \(N\)th-order filter "remembers" (i.e., the output can be calculated from) at most \(N\) past samples. The filter delays can be implemented with labeled storage locations, \(M_i\) and data transfers.

The program to implement the structure of Fig. 6(a) is straightforward. Everytime an input sample \(x_n\) is present in \(M_1\) the CPU needs to subtract \(M_1\) from \(M_5\), add the result to \(M_7\), add \(M_6\) to \(M_7\), copy \(M_6\) to an intermediate memory \(M_1\), shift \(M_1\) one to the right and add it to \(M_7\) (the addition of \(M_6\) and \(M_7\) with the addition of \(2^{-1}M_6\) to \(M_7\), implements the multiplication by 1.5). \(M_7\) will have the calculated \(y_n\), which can be converted to an analog signal with a D/A converter. Before finishing this cycle, we need to reproduce the effect of time and get ready to receive the next sample. Therefore, the data value stored in \(M_6\) must be moved to \(M_7\), \(M_5 \rightarrow M_6\), \(M_4 \rightarrow M_5\), \(M_3 \rightarrow M_4\), \(M_2 \rightarrow M_3\), \(M_1 \rightarrow M_2\), and wait for the next sample to be placed in \(M_1\) (assuming a real-time, interrupt driven application) in order to start the calculations again.

This filter is implemented in less than 50 µs in a TI 9900 microcomputer. The calculations are performed in fixed point arithmetic, without the need for a multiply instruction. So any microcomputer system (8 or 16 bits) can be utilized. However, one needs to remember that with an 8 bit microcomputer the precision is not as good—we can represent only 256 different values, that is to say, it is the same as making calculations with a \(2^8\) digit calculator.

Another related problem is overflow. If two positive integers are added in fixed point arithmetic, and the sum is more than 128 the result is a negative value due to the CPU's internal rules of number representation. Therefore, the input signal level must be controlled. This worsens the precision problem discussed earlier. The use of a 16 bit microcomputer plus a 10 or 12 bit A/D converter is recommended (or perform all the calculations in double precision arithmetic in an 8 bit microcomputer) to achieve a signal-to-noise ratio at the output of at least 40 dB [9], which is adequate for most biological signal processing applications.

VII. STOPBAND FILTER DESIGN IN EEG SLEEP STUDIES

In the analysis of sleep, the five prominent EEG activities are: delta (0.5–2 Hz), theta (2–6 Hz), alpha (8–12 Hz), sigma (12–16 Hz), and beta (16–30 Hz). The stopband filters utilized to process these activities according to the block diagram of Fig. 1 have transfer functions displayed in Table III.

Notice that the sampling frequencies utilized are not constant for all the EEG waves (they are submultiples of 1 kHz). This derives from the method used to estimate the frequency of the EEG waves which calls for a constant precision throughout the bands. The multirate sampling scheme implemented is described elsewhere [8], but it is worth mentioning that with careful selection of the sampling frequencies it is possible to realize all the filters with only two structures: a single 12th-order bandpass for alpha, sigma, and beta, and a single eighth-order low-pass for delta and theta.

As can be seen from Table III, the \(-3\) dB frequencies for each filter cover, but are wider than the frequencies of
### TABLE III
FILTERS AND SAMPLING FREQUENCIES FOR EEG SLEEP ANALYSIS

<table>
<thead>
<tr>
<th>EEG WAVES</th>
<th>SAMPLING FREQUENCY (Hz)</th>
<th>PASSBAND IMPLEMENTATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>27.7</td>
<td>$H_1(z) = (z^{-3}s_1)(z^{-2}s_1)(z^{-1}s_1)(z^{-2}s_2^{-1}s_2)$</td>
</tr>
<tr>
<td>0</td>
<td>63.3</td>
<td>$H_0(z) = H_1(z)$</td>
</tr>
<tr>
<td>0</td>
<td>166.6</td>
<td>$H_2(z) = (z^{-4}s_1)(z^{-2}s_1)(z^{-2}s_1.75s_1^{-1}s_1)(z^{-2}s_1)$</td>
</tr>
<tr>
<td>0</td>
<td>250.0</td>
<td>$H(z) = H_2(z)$</td>
</tr>
<tr>
<td>8</td>
<td>323.3</td>
<td>$H(z) = H_2(z)$</td>
</tr>
</tbody>
</table>

Fig. 10. EEG filter outputs for the delta, theta, and sigma bands in a typical sleep stage three epoch. Notice the difference in sampling frequencies apparent in the ladder like structure of the theta and delta waveforms.

VIII. Summary

A method is presented for designing linear phase digital filters with a finite impulse response. The filter passband is always an arcade of a sinusoidal function (or one of its powers). Therefore, only filters which are wide-band, which have corner frequencies not critically located and defined at $-3$ dB can be efficiently implemented. The filter stopband can be tightly controlled by placing additional zeros midway between stopband zeros. The time domain characteristics of these filters are very good because their impulse response is limited and the filters are linear phase.

In biological signal processing, due to the intrinsic variability of the signals, the important filter frequencies are not critically located. Therefore, this class of filters gives good results with small orders ($N \leq 14$) yielding structures which can be implemented in microcomputers even for real-time applications. Moreover, there is a certain de-
gree of freedom in choosing the sampling frequencies which can be utilized to design efficient multirate filtering schemes as is shown in [8].

REFERENCES


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