Analog Hardware Implementation of Adaptive Filter Structures

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Abstract

We have implemented a four-tap adaptive filter in a continuous-time analog VLSI circuit. Since an ideal delay is impossible to implement in continuous-time hardware, we implemented the delay line as a cascade of low-pass filters (called the Gamma filter). Since many years of research in our lab has shown that the Gamma filter outperforms the delay line for a wide range of applications, the Gamma filter should not be considered merely a crude approximation of the ideal delay line. We show measured results from an analog chip that solves the problem of system identification—identifying an unknown linear circuit from its input/output relationship. Furthermore, we believe that a cascade of all-pass filters (called the Laguerre filter) may potentially outperform the Gamma filter and we demonstrate a feedforward Laguerre filter still without adaptive weights.

I. Introduction

Transversal filters using ideal tap delay lines are widely used in adaptive systems. The standard digital transversal filter is shown in Figure 1a. The filter output is a weighted sum of delayed inputs. The weights are adapted using an optimization procedure that minimizes the mean square error between a desired signal and the filter output [15][2]. Some applications where these filters have attained considerable success include system identification, linear prediction, channel equalization and echo cancellation [15][2]. The reason for this success is, besides the simplicity of the transversal filter structure, the unimodality of its error surface, and the existence of fast and efficient adaptive algorithms to adjust its parameters. The principal problem with the transversal filter, which is also related to its advantage, is that its impulse response has a finite duration (it is an FIR filter). For this reason, when this filter is used to approximate a system with a long (possibly infinite) impulse response the minimum number of delays of the filter required to provide an acceptable approximation can be quite high. The problem can be partially solved using filters with an infinite impulse response (IIR filters). However, these filters have their own problems, especially if output error models are used. Among these are possible multimodal error surfaces, and possible instability problems related to the adaptation of the poles of these filters.

Since it is impossible to design an ideal delay line
in continuous-time hardware, many analog designers believe that the best that can be done is to try to "approximate" the ideal delays using a cascade of low-pass filters. Figure 1b shows such a strategy using a cascade of transconductance amplifiers and capacitors. Indeed, such a technique is shown in Mead's 1989 textbook [6]. We have studied exactly this structure (cascaded low-pass filters) as memory elements for adaptive filters and neural networks for many years. An irony is that this structure—called the Gamma filter—generally outperforms the ideal delay line with the same number of taps since the former provides a mechanism to let the network choose the most appropriate memory depth/resolution for the task at hand. This is easily done by adapting the memory depth using the output mean square error in training [14]. The Gamma memory is a marked improvement over the tap delay line because it has a free parameter that is able to modify the relative angle between the input signal and the memory space [8]. The Gamma model has been applied to a diverse set of real-world problems: echo cancellation [7], system identification [11] [9], time series prediction [4], noise reduction [5], and dynamic modeling [10]. In all cases it was established that the Gamma memory is superior to the tap delay line.

We have implemented the delay-line component of the Gamma filter exactly as shown in Figure 1b. Each stage consists of a transconductance amplifier connected as a follower with its output driving a capacitor—realizing a first-order low-pass filter with a 3 dB radian frequency equaling $1/\tau$. The CMOS transamp is operated within the subthreshold region so that a large dynamic range of $\tau$ can be obtained. For speech processing applications, the necessary dynamic range of $1/(2\pi\tau)$ is from 100 Hz to 10 KHz, which can be achieved by an exponentially controlled bias voltage.

The ideal and measured impulse tap impulse responses for each tap of the the Gamma filter are shown in Figure 2.

II. ADAPTATION

The power of transversal filters is their ability to adapt to a changing environment. The input-output relationship of a transversal filter with Gamma memory is given by

$$y(t) = \sum_{k=0}^{N} w_k(t)x_k(t)$$

(1)

Weight adaptation can be formulated as a parametric least-squares problem which accepts an iterative solution based on the LMS gradient descent method[15]. In order to adapt the weights $w_k$s, we use the following continuous-time gradient descent update:

$$\tau_w \cdot \frac{dw_k(t)}{dt} = e(t) \cdot x_k(t)$$

(2)

where $\tau_w$ (time constant of the weight update) $\gg \tau$. This dynamic equation requires basic primitives such
as adders, integrators and multipliers that we have efficiently implemented in analog VLSI using current summation, capacitor integration and Gilbert multipliers. Although gradient descent can still be used to adapt the time-scale parameter $\tau$, the equations are more difficult. They yield a nonconvex optimization problem and we do not address this problem here.

We have experimented with our continuous-time adaptive Gamma filter hardware for a simple system ID problem. The set up for system ID is shown in Figure 3a. In order to make the Gamma filter more resistant to consistent offsets in each stage, we have used the difference between adjacent taps as our input to the multiplier. Since we used a differential-input multiplier this was a very easy change. The circuitry is designed to minimize the square error between the output of the two filters. The unknown system is a discrete analog circuit designed to implement a standard

Sallen-Key low pass filter shown in Figure 3b [1]. This type of problem is typically solved with adaptive filters but rarely are continuous-time aspects considered.

The input to the system was a pseudo-random bit stream that was filtered to achieve a flat spectrum noise input. The system is set up so that the gamma filter will adapt its weights to minimize the mean square error between the filter output and the output of the unknown plant. Figure 4 shows a sample of the noise input as well as the desired output of the unknown plant.

Figure 5 shows the output of the filter during and after convergence. The gamma filter is not able to exactly match the Sallen-Key circuit because the two sys-
tems have different forms. However, the circuit does a good job in approximating the unknown plant.

Figure 6a shows the changes in the first weight during convergence for three different initial values. Figure 6b shows the path of the weight update for the two weights. As expected for a unimodal energy surface, that same optimal weight values are reached independent of the initial conditions. In order to accurately measure this data, we slowed the learning rate by applying large external capacitors to each weight. With these external capacitors, the convergence rate for this example is on the order of one minute. Much faster convergence can be reached by increasing the learning rate, however, as with the digital LMS algorithm, there is a tradeoff between the learning rate ($\tau_w$) and the variance of the weight values at convergence [15].

III. GENERALIZED FEEDFORWARD FILTERS

We can generalize the transversal filter as shown in Figure 7. Each delay of the transversal filter is replaced by a first-order filter $G(s)$. This preserves many of the properties of transversal filters and gives rise to continuous-time generalization of systems that have an infinite impulse response. There are other first-order filters besides the Gamma that produce acceptable approximations of systems with long impulse responses with a much smaller number of parameters than a transversal filter.

The Gamma filter is a special case of the generalized feedforward filter with

$$H(s) = \frac{1}{\tau s + 1} \quad \text{and} \quad G(s) = \frac{\tau s - 1}{\tau s + 1} \quad (3)$$

The Laguerre filter is another special case of the generalized filter shown in Figure 7 if we set the transfer functions to be

$$H(s) = \frac{1}{\tau s + 1} \quad \text{and} \quad G(s) = \frac{\tau s - 1}{\tau s + 1} \quad (4)$$

For the Laguerre, the first stage is a low pass filter, while subsequent stages are all-pass filters, each with a pole and zero in the same location as the pole for the first low-pass filter. Intuitively, we expect that the Laguerre memory will perform better than the Gamma memory because these Laguerre functions form an orthogonal set [12] [13]. We have successfully fabricated the Laguerre memory in 2 $\mu$m CMOS. Figure 8 shows the circuitry for a single all-pass stage as well as the measured outputs of each stage in a cascade for a sine wave input. The results show that the sine wave is delayed but not attenuated by the filter.

We simulated the continuous-time implementation of both the Gamma and Laguerre filters for a system identification problem. White noise is filtered by an unknown plant and an adaptive system is set up as shown in Figure 9 to identify the unknown transfer function. This type of problem is typically solved with adaptive filters but rarely are continuous-time aspects considered[3].

We show in Figure 10a, a comparison of the convergence rates for the system using the two different filters. As expected, the Laguerre shows a much faster convergence rate. However, in an analog implementation, we are not as much concerned with convergence speed as we are with the effects of noise and offsets in the system. We have added random noise with a certain standard deviation to the output of each tap to simulate the effects of real-world noise and random

1Technically speaking, the standard Laguerre filter requires that $w_0$ must be zero [12].
parameter variations. The results shown in Figure 10b indicate that the Laguerre is more robust the Gamma for the same amount of noise, making it a much better candidate for analog implementation.

IV. CONCLUSION

We have fabricated a four-tap Gamma filter in continuous-time analog VLSI. We have demonstrated the hardware performance on a real system identification problem. We plan to investigate circuitry for time-scale adaptation of the Gamma filter. We have also shown the implementation of the feedforward Laguerre filter which we expect to ultimately outperform the Gamma filter. We are currently fabricating a Laguerre filter with adaptive weights so that we can make direct comparisons between the two adaptive filters.

Fig. 8. Top, the all-pass circuit used to implement the Laguerre filter. Bottom, measured results from a fabricated all-pass circuit showing an input sine wave is delayed but not attenuated by a cascade of all-pass stages.

Fig. 9. Block diagram for system identification simulation

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Fig. 10. *Comparison of Gamma and Laguerre Results.* Top (a) shows the faster learning rate of the Laguerre vs. the Gamma. Bottom (b) shows the comparison of MSE vs. the amount of added random noise for the Laguerre and Gamma. The Laguerre shows a lower MSE for the same amount of added noise at the tap outputs.