THE EFFECT OF FILTERING IN THE EEG CORRELATION
DIMENSION ESTIMATION: EXPERIMENTAL RESULTS

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Abstract

This work describes the effect of linear filtering in the estimation of the correlation dimension of the electroencephalogram (EEG). Three types of filters, and several bandwidths were applied to both the signal and to white noise sequences, in order to measure experimentally the effect in the correlation dimension. The filter type does not seem to be important. The filter bandwidth must be judiciously set not to disturb the correlation dimension measurements. The filter bandwidth also affects the correlation map of white noise sequences (the correlation exponent fails to increase linearly with the embedding dimension).

Introduction

The study of the electroencephalogram (EEG), from the point of view of nonlinear dynamics, is a recent research area. The major reason to use still another technique to study the EEG signal is related to the partial success of the existing techniques, and to the never ending questions about brain dynamics. In this new approach the signal is considered the output of a nonlinear system, which we seek to characterize by studying the properties of its output, the EEG.

An efficient method to study nonlinear dynamical systems is to describe their state space properties. For the EEG the problem is compounded by the fact that no equation is available to describe the signal, i.e. it is necessary to estimate from the output all the system parameters. Moreover the signal is intrinsically noisy, therefore the study of the effect of the noise in the dimensionality analysis is a very important step. We have initiated an experimental study of the effect of parameters selection for the correlation dimension estimation [1]. The correlation dimension is one of the most popular measures to estimate the order of the singularities because it is robust, and much faster to implement than the other two most widely used order measures, the Hausdorff and the information dimension [2].

The correlation dimension is a lower bound to both the information dimension and the Hausdorff dimension, normally with very tight bounds. It measures the degree of spatial correlation between initially adjacent orbits that are time uncorrelated. The experimental parameters that affect the estimation of the correlation dimension are the segment length, the time lag and the set of values of embedding dimension for which the estimation is done. In [1], these aspects are experimentally studied to achieve robust order estimation.

It is important to recall that theoretically, the noise has a dimension that increases proportionally to the embedding dimension because, since white noise is uncorrelated, when the embedding dimension is increased, a proportional increase in the correlation dimension is observed, giving the traditional straight line in the correlation exponent map (CEM). This means that additive white noise will increase the estimation of the signal correlation dimension. In order to attenuate the noise, linear lowpass filtering is normally utilized. In this paper, the fundamental question to be addressed is what is the effect of linear filtering in the correlation estimation of the EEG signal and noise.

We will calculate the effect on the correlation dimension of white Gaussian noise when filtered by the same filter we use to clean the EEG signal from additive noise. In this way a much more realistic picture is achieved for the evidence of the "knee" in the correlation exponent map that attests spatial correlation, the hallmark of the existence of chaotic attractors in the EEG.

This aspect has been forgotten in the EEG correlation dimension literature [3, 4]. The only reference to it is given in [5], but it is inconclusive (the hypothesis of a chaotic/random component in the EEG is postulated). In the physics literature, Crutchfield and Farmer among others have shown the effect of noise in the obliteration of high periods and bifurcations, and the non zero value of Lyapunov exponents at these points.

Materials and Methods

Three different linear lowpass digital filter types (linear phase FIR, Butterworth, Chebyshev) with 9th order and different cutoff frequencies (0.16π, 0.4π and 0.64π), corresponding to 20, 50 and 80Hz were applied to the EEG and to pseudo random white noise (with periodicity much larger than the segment length utilized for this study). The segment length for sleep stage 2 EEG segment was set at 3750 samples, and the delay at 13 [1]. The same parameters were used for the white noise (the lag is not important for white noise, since the signal is uncorrelated). We calculate the correlation dimension by the following equation

\[ C(t) = \lim_{n \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} H(r - |x_i - x_j|) \]

where \(| |\) is the Euclidean distance, \(N\) the total number of points and \(H\) the Heaviside function.
Results

First the correlation dimension map for the EEG signal is obtained without filter and filtered respectively with the FIR, Butterworth and Chebyshev filters. Figure 1 shows the plot of the correlation exponent as a function of the embedding dimension for the theoretical white noise, the unfiltered EEG and the EEG filtered with the three filter types. The estimations of the correlation dimension for the three different filter types are extremely similar (see Table I). However, there is a difference with respect to the unfiltered EEG, which shows a dimension of 6.56. We conclude that a decrease in bandwidth affects the correlation dimension estimation. Since the EEG does not have appreciable energy above 50 Hz, the signal should be prefiltered with a lowpass filter with this cutoff frequency for an estimation that discards any noise above this value (such the 60 Hz interference). The filter type does not seem to be important for the estimation.

TABLE I

<table>
<thead>
<tr>
<th>SIGNAL</th>
<th>CORRELATION DIMENSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>original</td>
<td>6.58 +/- 0.09</td>
</tr>
<tr>
<td>Butterworth80</td>
<td>4.92 +/- 0.06</td>
</tr>
<tr>
<td>Butterworth50</td>
<td>4.71 +/- 0.08</td>
</tr>
<tr>
<td>Butterworth20</td>
<td>4.45 +/- 0.06</td>
</tr>
<tr>
<td>Chebyshev50</td>
<td>4.74 +/- 0.08</td>
</tr>
<tr>
<td>FIR50</td>
<td>4.55 +/- 0.07</td>
</tr>
</tbody>
</table>

Now we are ready to study the result of filtering in the correlation exponent map of the noise. Since the filter type is not relevant for the estimation, we used the Butterworth filter. We changed the cutoff filter of the lowpass filter from 20, 50 and 80 Hz as before. The plot of the correlation dimension versus the embedding dimension of the white noise sequence and its filtered versions is depicted in Figure 2. It is obvious from the plot of the filtering in the correlation exponent map. For the white noise sequence, no knee is visible, which means that the sequence can in fact be considered totally uncorrelated. When more severe filtering is applied, correlations show up in the CEM, although a clear knee is not visible at least for embedding dimensions up to 16. This is apparent in all the curves because the correlation exponent does not increase proportionally to the embedding dimension. However, embedding dimension 16 the correlation exponent is 12 for the 50 Hz filtered noise, but only 8 for the 20 Hz filtered noise.

These results show unquestionably that a reduction in bandwidth does affect the correlation exponent map of white noise sequences. Therefore, the normal way of displaying the correlation dimension of the EEG signal, comparing it to the theoretical straight line of white noise is misleading. We propose that the plot of the correlation exponent (of the EEG or any other band-limited signal) should be graphed with the correlation exponent of white noise for the same measurement bandwidth, as Figure 3 shows.

References


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