Problem I

An “unknown” plant has transfer function \( H(z) = \frac{1-z^{-10}}{1-z^{-1}} \) and its output is added with white Gaussian noise of power \( N=0.1 \). The input to the plant is alpha stable noise with \( \alpha=1.5 \). To generate this noise use the characteristic function \( \varphi(t) = \exp(-\gamma |t|^\alpha) \) with \( \alpha=1.5 \) and choose \( \gamma=1 \). Generate 5,000 samples of the alpha stable noise as well as the white Gaussian noise.

The user has only access to the noisy output of the plant and to its input. The goal of this problem is to design a Wiener filter to identify the unknown plant transfer function. You can NOT use the fact that you know the plant to design the Wiener filter, but you can use this knowledge to validate the solution obtained. Use the normalized MSE as the quality of the identification (normalize by the power of the input). I suggest that you use filters of order 5, 15 and 30, and windows of size 100, 500 samples to estimate the autocorrelation function and cross correlation vector. Compare the accuracy of the system identification by computing the weight error power.

\[
WSNR = 10 \log \left( \frac{w^*^T w^*}{(w^* - w(n))^T (w^* - w(n))} \right)
\]

where \( w^* \) is the optimal weight vector from the plant, and \( w(n) \) means the calculated Wiener parameters for the window of data (if you use different windows you will find that these parameters slightly vary).

Show the effect of increasing the noise \( N \) (\( N=0.3, 0.5 \)) from your experiments both in terms of MSE and WSNR. Explain what you observe.

Problem II

Solve the above problem using the LMS algorithm. Show the learning curve and estimate the misadjustment. Use three different stepsizes to illustrate the compromise between speed of adaptation and misadjustment. Compare with Problem I in terms of MSE, WSNR and computational complexity.