The purpose of this homework is to let you program the backpropagation through time algorithm to train recurrent networks. The problem is to create an oscillator that will create a figure 8 in 2D space by learning the patterns in prediction. Generate a sequence of samples in 2D space \{x(i),y(i)\} where \(x(i)\) is a triangular wave of amplitude 1/\(-1\) and period 64 samples, while \(y(i)\) is a sinewave of amplitude 1 and period 32 samples.

The network architecture is a recurrent MLP, i.e. an input layer with two PEs, one hidden layer MLP with 15 PEs, and two outputs but where the hidden PEs are fully connected (except for self-feedback – see figure).

a) You will train the system as a predictor. Once trained you will fix the weights, use the initial condition from \(x(1),y(1)\) and feedback the system output back to the input thru a delay. If the system learned the pattern perfectly it will recreate it forever! Of course this will never happen! So please compute the MSE during the 10 trajectory and the original. Repeat for the 500th trajectory and you will see a slight degradation over time.

One of the difficulties is that you are using the system in a way that is different from the way it was trained. In fact, you have used the true input all the time during training, didn’t you? When the system is trained this way it does not work that well as an oscillator as you have seen above.

b) Now do the following. Train the system as in a) for 100 steps. Then continue training by disconnecting the input and using the output of the system delayed by one step. The desired response will be the same as in a). Monitor the MSE and stop training when the error is small. Then test as before.

Compare the performance with a).

You can complicate the trajectory by halfing the period of the sinewave and create 4 loops.