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NORMALIZED LMS

The normalized LMS can be derived from the mathematical *principle of minimal disturbance*: "find the stepsize that disturbes the weights the least but still converges".

As you can expect this will lead to a constrained optimization problem.

In fact, define the disturbance as the Euclidean distance between the current and next weights

$$\|\Delta \bar{w}(n+1)\|^2$$

$$\Delta \bar{w}(n+1) = \bar{w}(n+1) - \bar{w}(n)$$

And make sure that the filter still converges

$$\bar{w}(n+1)\,\bar{x}(n) = d(n)$$

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To solve this problem, use the method of the Lagrange multipliers

The optimization can be captured in the cost function

$$J(n) = \| \Delta W(n+1) \|^2 + \lambda \left(d(n) - \bar{w}(n+1)\,\bar{x}(n) \right)$$

where λ is a constant called the Lagrange multiplier

To solve take the derivative and equate it to zero.

$$\frac{\partial J(n)}{\partial \bar{w}(n+1)} = 2\left(\bar{w}(n+1) - \bar{w}(n) \right) - \lambda \bar{x}(n) = 0$$

Solve for the unknown λ $\Longrightarrow \bar{w}(n+1) = \bar{w}(n) + \frac{1}{2}\lambda \bar{x}(n)$

$$d(n) = \bar{w}(n+1)\,\bar{x}(n) = \bar{w}(n)\,\bar{x}(n) + \frac{1}{2}\lambda \| \bar{x}(n) \|^2$$

$$\Longrightarrow \lambda = \frac{2e(n)}{\| x(n) \|^2}$$

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Now find out what is the minimal increment.

$$\Delta \bar{w}(n+1) = \bar{w}(n+1) - \bar{w}(n)$$

$$= \frac{1}{\|x(n)\|^2} \bar{x}(n) e(n)$$

Normally we use a stepsize for user control.

$$\bar{w}(n+1) = w(n) + \frac{\eta}{\|\bar{x}(n)\|^2} \bar{x}(n) e(n)$$

Note that the stepsize essentially becomes time varying controlled by the power of the input.

The NLMS becomes also INDEPENDENT of the input power, which is very useful in practice.

To avoid instability in low amplitude portions of the input, we "regularize" the denominator (effectively imposing the maximal stepsize for the application).

$$\bar{w}(n+1) = \bar{w}(n) + \frac{\eta}{\delta + \|x(n)\|^2} \bar{x}(n) e(n)$$

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NLMS STABILITY

The iteration will converge in the mean square error sense if

$$\| w(n) - w^* \|^2 = \| v(n) \|^2 \longrightarrow 0$$

By substitution we can show that

$$D_v(n+1) - D_v(n) = \eta^2 E\left[\frac{|e(n)|^2}{\|x(n)\|^2}\right] - 2E\left[\frac{\bar{v}(n)\bar{x}(n)e(n)}{\|x(n)\|^2}\right]$$

So for convergence

$$0 < \eta < 2\frac{E[\bar{v}(n)\bar{x}(n)e(n)]}{E[|e(n)|^2]}$$

The optimal value is obtained when

$$\eta_{OPT} = \frac{E[\bar{v}(n)\bar{x}(n)e(n)]}{E[|e(n)|^2]}$$