

20 minutes close book

By signing my name below, I declare that I have not given or received help from others during the quiz.

Name: SOLUTION, ID #: _____

Please answer in the space provided (1 question)

Question 1: A LTI system has frequency response

$$H(e^{j\hat{\omega}}) = (1 - e^{j(0.25\pi - \hat{\omega})})(1 - e^{j(-0.25\pi - \hat{\omega})})(2 + e^{-j\hat{\omega}})$$

The input to the system is

$$x(n) = -1 + 3\cos(0.5\pi n) + 2\delta(n+1)$$

Determine the corresponding output $y(n)$ for $-\infty < n < \infty$.

$$(1 - e^{j(0.25\pi - \hat{\omega})})(1 - e^{j(-0.25\pi - \hat{\omega})}) = 1 - (e^{j0.25\pi} + e^{-j0.25\pi})e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} = 1 - 2\cos\frac{\pi}{4}e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 2 + (1 - 4\cos\frac{\pi}{4})e^{-j\hat{\omega}} + 2(1 - 2\cos\frac{\pi}{4})e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} = 2 - 1.8e^{-j\hat{\omega}} + 0.5e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}}$$

$$\text{So } h(n) = 2\delta(n) - 1.8\delta(n-1) + 0.5\delta(n-2) + \delta(n-3)$$

Now $y(n) =$ write $x(n) = x_1(n) + x_2(n) + x_3(n) + x_4(n) = -1 + \frac{3}{2}e^{j0.5\pi n} + \frac{3}{2}e^{-j0.5\pi n} + 2\delta(n+1)$

$$y_1(n) = -1(H(e^{j\hat{\omega}})|_{\hat{\omega}=0}) = -(2 - 1.8 + 0.5 + 1) = -1.7$$

$$y_2(n) = (H(e^{j\hat{\omega}})|_{\hat{\omega}=0.5\pi}) \cdot \frac{3}{2}e^{j0.5\pi n} = (2 - 1.8e^{-j\frac{\pi}{2}} + 0.5e^{-j\pi} + e^{j1.5\pi}) \cdot \frac{3}{2}e^{j\frac{\pi}{2}n} = \frac{3}{2}e^{j\frac{\pi}{2}n} (1.4 - j0.8)$$

$$y_3(n) = \frac{3}{2}e^{-j\frac{\pi}{2}n} (1.4 + j0.8) = \frac{3}{2}e^{-j\frac{\pi}{2}n} (1.6e^{j0.16\pi})$$

$$y_4(n) = 4\delta(n+1) - 3.6\delta(n) + 1.2\delta(n-1) + 2\delta(n-2)$$

$$\text{So } y(n) = -1.7 + 4.8\cos\left(\frac{\pi}{2}n + 0.16\pi\right) + 4\delta(n+1) - 3.6\delta(n) + 1.2\delta(n-1) + 2\delta(n-2)$$