EEL 3135 – Quiz 4
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20 minutes close book

By signing my name below, I declare that I have not given or received help from others during the quiz.

Name: _______________________, ID #: _______________________

Please answer in the space provided (1 question)

Question 1: A LTI system has frequency response

\[ H(e^{j\omega}) = (1 - e^{j(0.25\pi - \omega)}) (1 - e^{j(-0.25\pi - \omega)}) (2 + e^{-j\omega}) \]

The input to the system is

\[ x(n) = -1 + 3\cos(0.5\pi n) + 2\delta(n+1) \]

Determine the corresponding output \( y(n) \) for \(-\infty < n < \infty \).

\[
\begin{align*}
(1 - e^{j(0.25\pi - \omega)}) (1 - e^{j(-0.25\pi - \omega)}) & = 1 - (e^{j\omega} + e^{-j\omega}) + e^{j\omega} e^{-j\omega} = \\
& = 1 - 2\cos\frac{\pi}{4} e^{j\omega} + e^{-j\omega} = 2 - 1.8 e^{j\omega} + 0.8 e^{-j\omega} = \\
H(e^{j\omega}) & = 2 + (1 - 4\cos\frac{\pi}{4}) + 2(1 - e^{j\omega} + e^{-j\omega}) = 2 - 1.8 e^{j\omega} + 0.8 e^{-j\omega} \\
\end{align*}
\]

So \( h(n) = 2\delta(n) - 1.8\delta(n-1) + 0.8\delta(n-2) + \delta(n-3) \)

Now \( y(n) = w(n) \cdot h(n) \)

\[ x(n) = x_1(n) + x_2(n) + x_3(n) + x_4(n) = -1 + \frac{3}{2} \epsilon + \frac{3}{2} \epsilon + 2\delta(n+1) \]

\[ y_1(n) = -1 \left( H(e^{j\omega}) \bigg|_{\omega = 0} \right) = - (2 - 1.8 + 0.5 + 1) = -1.7 \]

\[ y_2(n) = \left( H(e^{j\omega}) \bigg|_{\omega = 0.5\pi} \right) \cdot \frac{3}{2} = (2 - 1.8 e^{j0.5\pi} + 0.8 e^{-j0.5\pi}) = \frac{3}{2} e^{j0.75\pi} \left( 1.4 e^{j0.8\pi} \right) \]

\[ y_3(n) = \frac{3}{2} e^{-j\frac{\pi}{2}} (1.4 + j0.8) = \frac{3}{2} e^{-j\frac{\pi}{2}} (1.6 e^{j0.8\pi}) \]

\[ y_4(n) = 3.6 \delta(n) + 1.2 \delta(n-1) + 2 \delta(n-2) \]

So \( y(n) = -1.7 + 4.8 \cos(\frac{\pi}{2} n + 0.16) + 4 \delta(n+1) - 3.6 \delta(n) + 1.2 \delta(n-1) + 2 \delta(n-2) \)