

SOLUTION

EEL 3135 - Quiz 2

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20 minutes close book

By signing my name below, I declare that I have not given or received help from others during the quiz.

Name: _____, ID #: _____

Please answer in the space provided (1 question)

Question 1: Compute the spectrum (Fourier coefficients a_k) of the periodic signal (period T_0)

$$x(t) = \begin{cases} \cos\left(\frac{8\pi}{T_0}t\right) & -T_0/4 \leq t \leq T_0/4 \\ 0 & \text{otherwise in } -T_0/2 \leq t \leq T_0/2 \end{cases}$$

$$a_k = \frac{1}{T_0} \int_{-T_0/4}^{T_0/4} \cos\left(\frac{8\pi}{T_0}t\right) e^{-j\frac{2\pi k t}{T_0}} dt = \cos\frac{8\pi t}{T_0} = \frac{e^{j\frac{8\pi t}{T_0}} + e^{-j\frac{8\pi t}{T_0}}}{2}$$
$$= \frac{1}{2T_0} \int_{-T_0/4}^{T_0/4} e^{j\frac{2\pi t}{T_0}(4-k)} dt + \frac{1}{2T_0} \int_{-T_0/4}^{T_0/4} e^{-j\frac{2\pi t}{T_0}(4+k)} dt$$

A B

$$A = \frac{1}{2T_0} \left(\frac{e^{j\frac{2\pi t}{T_0}(4-k)}}{j\frac{2\pi}{T_0}(4-k)} \right) \Bigg|_{-T_0/4}^{T_0/4} = \frac{1}{2\pi(4-k)} \sin \frac{\pi}{2}(4-k)$$

$$B = \frac{1}{2T_0} \left(\frac{e^{-j\frac{2\pi t}{T_0}(4+k)}}{-j\frac{2\pi}{T_0}(4+k)} \right) \Bigg|_{-T_0/4}^{T_0/4} = \frac{1}{2\pi(4+k)} \sin \frac{\pi}{2}(4+k)$$

$$a_k = \frac{1}{2\pi(4-k)} \sin \frac{\pi}{2}(4-k) + \frac{1}{2\pi(4+k)} \sin \frac{\pi}{2}(4+k)$$