PROBLEM 8.1:

$$y[n] = \sqrt{2}\,y[n-1] - y[n-2] + x[n]$$

$x[n] = \delta[n]$

"At rest" condition $\Rightarrow$ $y[n]=0$ for $n<0$.

$$y[0] = \sqrt{2}\,y[-1] - y[-2] + x[0] = (\sqrt{2})0 - 0 + 1 = 1$$
$$y[1] = \sqrt{2}\,y[0] - y[-1] + x[1] = (\sqrt{2})1 - 0 + 0 = \sqrt{2}$$
$$y[2] = \sqrt{2}\,y[1] - y[0] + x[2] = (\sqrt{2})\sqrt{2} - 1 + 0 = 1$$
$$y[3] = (\sqrt{2})1 - \sqrt{2} + 0 = 0$$
$$y[4] = (\sqrt{2})0 - 1 + 0 = -1$$

The general formula is

$$y[n] = A_1(r_1)^n + A_2(r_2)^n \qquad \text{for } n \geq 0$$

where r_1 & r_2 are the poles.

$$H(z) = \frac{1}{1 - \sqrt{2}z^{-1} + z^{-2}}$$

Poles are roots of denominator:

$$\frac{\sqrt{2} \pm \sqrt{2-4}}{2} = \frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2} = e^{\pm j\pi/4}$$

$$y[n] = A_1(e^{j\pi/4})^n + A_2 e^{-j\frac{\pi}{4}n}$$

Now, we evaluate A_1 & A_2 from known values of $y[n]$. We use $n=2$ and $n=4$

$$y[2] = 1 = A_1 e^{j\pi/2} + A_2 e^{-j\pi/2} = jA_1 - jA_2$$
$$y[4] = -1 = A_1 e^{j\pi} + A_2 e^{-j\pi} = -A_1 - A_2$$

Solve the simultaneous equations:

$$1-j = -2jA_2 \quad \text{and} \quad 1+j = 2jA_1 \Rightarrow A_1 = \frac{1+j}{2j} = \frac{1}{2} - j\frac{1}{2}$$

$$A_2 = A_1^* \qquad A_1 = \frac{\sqrt{2}}{2}e^{-j\pi/4}$$

$$y[n] = \frac{\sqrt{2}}{2}e^{-j\pi/4}e^{j\frac{\pi}{4}n} + \frac{\sqrt{2}}{2}e^{+j\pi/4}e^{-j\pi n/4} \qquad \underline{\text{for } n \geq 0}$$

$$= \sqrt{2}\cos\left(\frac{\pi}{4}n - \frac{\pi}{4}\right) = \sqrt{2}\cos\left(\frac{\pi}{4}(n-1)\right)$$

PROBLEM 8.3:

$h[n] = 5(0.8)^n u[n]$ $\;\;$ & $\;\;$ $x[n] = \delta[n] - \alpha\delta[n-5]$

$$y[n] = h[n] * x[n] = 5(0.8)^n u[n] - 5\alpha(0.8)^{n-5} u[n-5]$$

Want $y[n] = 0$ for $n \geq 5$

$$\Rightarrow \quad 5(0.8)^n - 5\alpha(0.8)^{n-5} = 0$$

$$\Rightarrow \quad 5 = 5\alpha(0.8)^{-5}$$

$$\Rightarrow \quad \alpha = (0.8)^5 \cong 0.3277$$

PROBLEM 8.5:

$$y[n] = \tfrac{1}{2}y[n-1] - \tfrac{1}{3}y[n-2] - x[n]$$

$$Y(z) = \tfrac{1}{2}z^{-1}Y(z) - \tfrac{1}{3}z^{-2}Y(z) - X(z)$$

$$\left(1 - \tfrac{1}{2}z^{-1} + \tfrac{1}{3}z^{-2}\right)Y(z) = -X(z)$$

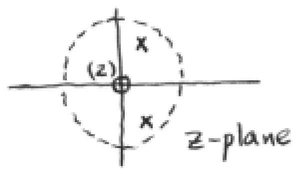
$$H(z) = \frac{Y(z)}{X(z)} = \frac{-1}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}}$$

Change to positive powers of z when finding poles and zeros.

$$H(z) = \frac{-z^2}{z^2 - \frac{1}{2}z + \frac{1}{3}}$$

poles are at
$z = 0.25 \pm j0.52$
$= 0.5774e^{\pm j0.357\pi}$
ANGLE $= \pm 64.34°$
or ± 1.123 rads

Numerator is z^2, so we have two zeros at $z=0$.



$$y[n] = \tfrac{1}{2}y[n-1] - \tfrac{1}{3}y[n-2] - x[n-2]$$

$$H(z) = \frac{-z^{-2}}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}} = \frac{-1}{z^2 - \frac{1}{2}z + \frac{1}{3}}$$ ← Same poles

If we take $\lim_{z\to\infty} H(z)$ we get $H(z) \to \frac{1}{z^2}$ so we have 2 zeros at $z = \infty$

$$y[n] = \tfrac{1}{2}y[n-1] - \tfrac{1}{3}y[n-2] - x[n-4]$$

$$H(z) = \frac{-z^{-4}}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}} = \frac{-1}{z^2\left(z^2 - \frac{1}{2}z + \frac{1}{3}\right)}$$

Now $H(z) \to 1/z^4$ as $z \to \infty$, so we have 4 zeros at $z = \infty$

We have 4 poles. The same two as above, plus 2 more poles at $z = 0$.

PROBLEM 8.9:

$$y[n] = -0.9\,y[n-6] + x[n]$$

(a) $Y(z) = -0.9 z^{-6} Y(z) + X(z)$

$$H(z) = \frac{1}{1+0.9z^{-6}} = \frac{z^6}{z^6+0.9}$$

SIX ZEROS AT $z=0$

(b) Poles are found as the solutions to

$$z^6 + 0.9 = 0$$

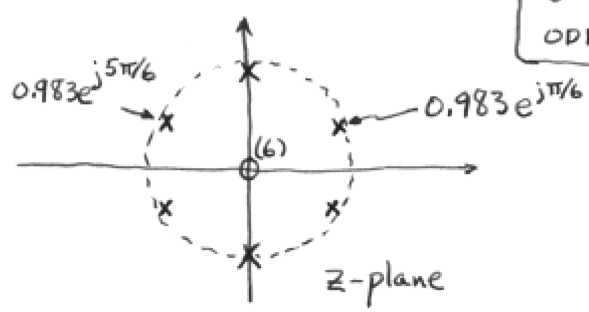
This involves the "roots of unity"

$$z^6 = -0.9 = 0.9\,e^{j\pi}e^{j2\pi\ell} \qquad \ell = 0,1,2,3,4,5$$

$$z = \sqrt[6]{0.9}\; e^{j\pi/6} e^{j\pi\ell/3}$$

$$= 0.983\, e^{j\pi(2\ell+1)/6}$$

ANGLES ARE:
$\frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$
ODD MULTIPLES of 30°



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

PROBLEM 8.14:

Characterize each system ($S_1 \to S_7$)

S_1: $H_1(z) = \dfrac{\frac{1}{2} + \frac{1}{2}z^{-1}}{1 - 0.9z^{-1}}$ $\Rightarrow$ pole at $z = 0.9$, zero at $z = -1$

$H_1(e^{j\hat{\omega}})$ is a LPF with a null at $\hat{\omega} = \pi$.

S_2: $H_2(z) = \dfrac{9 + 10z^{-1}}{1 + 0.9z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$, zero at $z = -10/9$

$H_2(e^{j\hat{\omega}})$ is an all-pass filter

S_3: $H_3(z) = \dfrac{\frac{1}{2}(1 - z^{-1})}{1 + 0.9z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$, zero at $z = 1$

$H_3(e^{j\hat{\omega}})$ is a HPF with a null at $\hat{\omega} = 0$.

S_4: $H_4(z) = \frac{1}{4}(1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})$

$= \frac{1}{4}(1 + z^{-1})^4$ $\Rightarrow$ 4 zeros at $z = -1$

$H_4(e^{j\hat{\omega}})$ is a LPF with null at $\hat{\omega} = \pi$.

DC value: $H_4(e^{j0}) = 4$.

S_5: $H_5(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} = \dfrac{1 + z^{-5}}{1 + z^{-1}}$

has 4 zeros around the unit circle.

No zero at $z = -1$; others at $e^{j(2\pi k/5 - \pi/5)}$

$H_5(e^{j\hat{\omega}})$ is a HPF with nulls at $\hat{\omega} = \pm\frac{\pi}{5}, \pm\frac{3\pi}{5}$

S_6: $H_6(z) = 1 + z^{-1} + z^{-2} + z^{-3} = \dfrac{1 - z^{-4}}{1 - z^{-1}}$

has 3 zeros around the unit circle at $z = \pm j, -1$

$H_6(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{2}, \pi$

S_7: $H_7(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} = \dfrac{1 - z^{-6}}{1 - z^{-1}}$

has 5 zeros around the unit circle at $z = e^{j\pi k/3}$

$H_7(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}, \pi$

(A) S_1 (C) S_6 (E) S_5

(B) S_3 (D) S_2 (F) S_4

PROBLEM 8.19:

Multiply out $H(z)$

$$H(z)=\frac{(1-z^{-1})(1-jz^{-1})(1+jz^{-1})}{(1-0.9e^{j2\pi/3}z^{-1})(1-0.9e^{-j2\pi/3}z^{-1})}$$

$$=\frac{(1-z^{-1})(1+z^{-2})}{1-2(0.9)\cos(2\pi/3)z^{-1}+(0.9)^2z^{-2}}$$

$$=\frac{1-z^{-1}+z^{-2}-z^{-3}}{1-0.9z^{-1}+0.81z^{-2}}$$

(a) Use the numerator & denominator polynomial coefficients as filter coefficients:

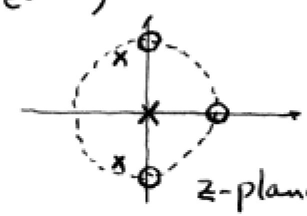
$$y[n]=0.9y[n-1]-0.81y[n-2]+x[n]-x[n-1]+x[n-2]-x[n-3]$$

(b) Multiply numerator & denominator by z^3:

$$H(z)=\frac{(z-1)(z-j)(z+j)}{z(z-0.9e^{j2\pi/3})(z-0.9e^{-j2\pi/3})}$$

Zeroes: $z=1$, j and $-j$

Poles: $z=0$, $z=0.9e^{\pm j2\pi/3}$



(c) The zeros of the numerator polynomial are on the unit circle at $z=e^{j0}$, $z=e^{j\pi/2}$ and $z=e^{-j\pi/2}$

When $x[n]=Ae^{j\varphi}e^{j\hat{\omega}n}$, the output $y[n]$ is

$$y[n]=H(e^{j\hat{\omega}})\cdot Ae^{j\varphi}e^{j\hat{\omega}n}$$

There the output will be zero when $H(e^{j\hat{\omega}})=0$.

That is, for $\hat{\omega}=0$, $\hat{\omega}=\pi/2$ and $\hat{\omega}=-\pi/2$.