PROBLEM 7.4:

(a) use filter coeffs: $H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$

(b) Use positive powers to extract poles and zeros

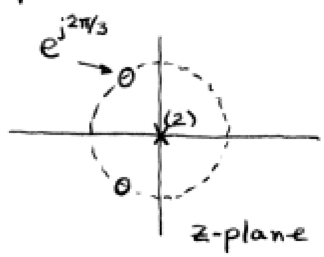
$$H(z) = \frac{1}{z^2}\left(\frac{1}{3}z^2 + \frac{1}{3}z + \frac{1}{3}\right)$$

TWO POLES AT $z=0$

zeros at

$$z = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

zeros: $1e^{\pm j2\pi/3}$



(c) $\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$

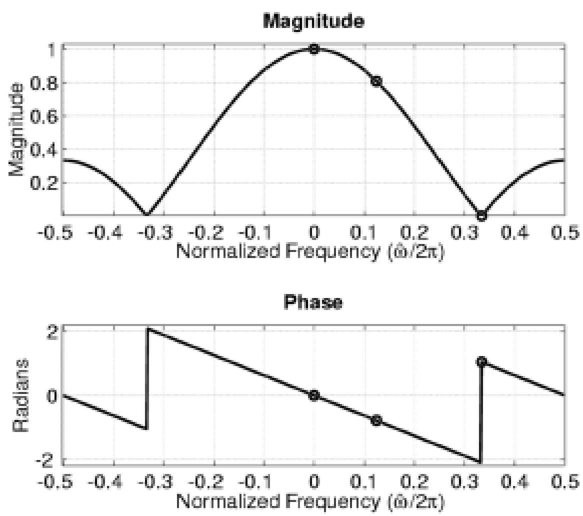
$$= \frac{1}{3} + \frac{1}{3}e^{-j\hat{\omega}} + \frac{1}{3}e^{-j2\hat{\omega}} = \frac{1}{3}e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}\right)$$

$$= e^{-j\hat{\omega}}\left(\frac{1+2\cos\hat{\omega}}{3}\right)$$

ANOTHER FORMULA:

$$\boxed{\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}\left(\frac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)}\right)}$$

(d) use MATLAB



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(e) Use Linearity & Frequency response at $\hat{\omega}=0$, $\hat{\omega}=\pi/4$ and $\hat{\omega}=2\pi/3$. These are marked on the plots of the frequency response.

$$y[n] = 4\mathcal{H}(0) + |\mathcal{H}(\pi/4)|\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{4} + \angle\mathcal{H}(\pi/4)\right)$$
$$-3\underbrace{|\mathcal{H}(2\pi/3)|}_{=0}\cos\left(\tfrac{2\pi}{3}n + \angle\mathcal{H}(2\pi/3)\right)$$

$$\mathcal{H}(0) = \tfrac{1}{3}+\tfrac{1}{3}+\tfrac{1}{3} = 1$$

$$\mathcal{H}(\pi/4) = e^{-j\pi/4}(1+2\sqrt{2}/2)/3 = \frac{1+\sqrt{2}}{3}e^{-j\pi/4} = 0.8047e^{-j\pi/4}$$

$\mathcal{H}(2\pi/3) = 0$ because $H(z)=0$ at $z = e^{\pm j2\pi/3}$

$$\therefore\ y[n] = 4 + 0.8047\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{2}\right)$$

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PROBLEM 7.5:

(a) $H(z) = (1-z^{-1})\underbrace{(1-jz^{-1})(1+jz^{-1})}_{1+z^{-2}}\underbrace{(1-0.9e^{j\pi/3}z^{-1})(1-0.9e^{-j\pi/3}z^{-1})}_{1-\underbrace{1.8\cos(\pi/3)}_{0.9}z^{-1}+0.81z^{-2}}$

$$H(z) = (1-z^{-1}+z^{-2}-z^{-3})(1-0.9z^{-1}+0.81z^{-2})$$

$$= 1-1.9z^{-1}+2.71z^{-2}-2.71z^{-3}+1.71z^{-4}-0.81z^{-5}$$

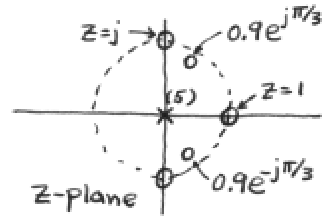
Use polynomial coeffs as filter coeffs:

$$y[n] = x[n]-1.9x[n-1]+2.71x[n-2]-2.71x[n-3]+1.71x[n-4]-0.81x[n-5]$$

(b) $H(z) = \frac{1}{z^5}(z-1)(z-j)(z-(-j))(z-0.9e^{j\pi/3})(z-0.9e^{-j\pi/3})$

FIVE POLES AT $z=0$

THE FACTORED FORM GIVES ALL THE ZEROS



(c) The zeros on the unit circle will cause nulling of $x[n] = Ae^{j\varphi}e^{j\hat{\omega}n}$

$z=1=e^{j0}$ so $\hat{\omega}=0$ is nulled

$z=j=e^{j\pi/2}$ so $e^{j\frac{\pi}{2}n}$ is nulled

$z=-j=e^{-j\pi/2}$ so $e^{-j\frac{\pi}{2}n}$ is nulled.

PROBLEM 7.6:

(a) $Y_1(z) = H_1(z)X(z)$

$$Y(z) = H_2(z)Y_1(z) = H_2(z)\big(H_1(z)X(z)\big)$$

$$= \underbrace{\big(H_2(z)H_1(z)\big)}_{H(z)} X(z) \quad \text{because } H(z) = \frac{Y(z)}{X(z)}$$

(b) Since $H_2(z)H_1(z) = H_1(z)H_2(z)$ because $H_1(z)$ and $H_2(z)$ are scalar functions.

$$\Rightarrow Y(z) = H_1(z)\underbrace{H_2(z)X(z)}_{\text{means that } H_2(z) \text{ is applied first}}$$

(c) $H_1(z) = \frac{1}{3}(1+z^{-1}+z^{-2})$ by using the filter coeffs.

$$H(z) = H_2(z)H_1(z)$$

$$= \tfrac{1}{3}(1+z^{-1}+z^{-2})\cdot\tfrac{1}{3}(1+z^{-1}+z^{-2})$$

$$= \tfrac{1}{9}(1+2z^{-1}+3z^{-2}+2z^{-3}+z^{-4})$$

(d) Convert to difference equation (i.e., filter coeffs)

$$y[n] = \tfrac{1}{9}\big(x[n]+2x[n-1]+3x[n-2]+2x[n-3]+x[n-4]\big)$$

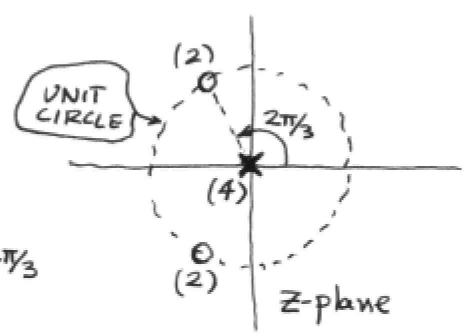
(e) Find the poles & zeros of $H_2(z)$, then "double" them because $H_1(z) = H_2(z)$.

$$H_2(z) = \tfrac{1}{3}z^{-2}(z^2+z+1)$$

$\frac{1}{z^2}$ contributes two poles at $z=0$

zeros are:

$$\frac{-1\pm\sqrt{1-4}}{2} = -\tfrac{1}{2}\pm j\tfrac{\sqrt{3}}{2} = e^{\pm j2\pi/3}$$



PROBLEM 7.6 (more):

(f)

$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$$

$$= \frac{1}{9}\left(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right)^2$$

$$= \frac{1}{9}e^{-j2\hat{\omega}}\left(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}\right)^2$$

$$= \frac{1}{9}e^{-j2\hat{\omega}}\left(1 + 2\cos(\hat{\omega})\right)^2$$

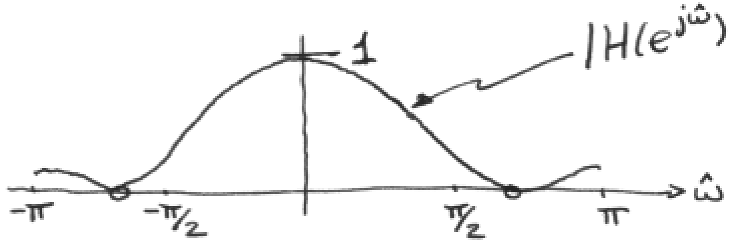
$$|H(e^{j\hat{\omega}})| = \frac{1}{9}\left(1 + 2\cos(\hat{\omega})\right)^2$$

At $\hat{\omega} = 0$, $|H| = \frac{1}{9}(3)^2 = 1$

At $\hat{\omega} = \pi/2$, $|H| = \frac{1}{9}(1)^2 = 1/9$

At $\hat{\omega} = 2\pi/3$, $|H| = 0$ because there is a zero on the unit circle.

At $\hat{\omega} = \pi$, $|H| = \frac{1}{9}(1-2)^2 = 1/9$



PROBLEM 7.12:

(a)
$$H(z) = (1 - z^{-1})(1 + z^{-2})(1 + z^{-1})$$
MULTIPLY OUTER FACTORS
$$= (1 - z^{-2})(1 + z^{-2}) = 1 - z^{-4}$$
$$\therefore \quad y[n] = x[n] - x[n-4]$$

(b) $H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = 1 - e^{-j4\hat{\omega}}$

(c) $H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}\left(e^{+j2\hat{\omega}} - e^{-j2\hat{\omega}}\right)$
$$= 2j\, e^{-j2\hat{\omega}} \sin 2\hat{\omega} = (2\sin 2\hat{\omega})\, e^{j(\pi/2 - 2\hat{\omega})}$$

MAG: $2\sin 2\hat{\omega}$ ALTHOUGH THIS HAS A SIGN CHANGE FOR $\hat{\omega} < 0$

PHASE: $\pi/2 - 2\hat{\omega}$

(d) BLOCK WHEN $H(e^{j\hat{\omega}}) = 0$

$\therefore$ SOLVE $2\sin 2\hat{\omega} = 0$
$$\Longrightarrow \quad \hat{\omega} = 0, \pi/2, \pi, -\pi/2$$

(e) Need $H(e^{j\pi/3})$ because that is the frequency of the input.

$$H(e^{j\pi/3}) = \left(2\sin \tfrac{2\pi}{3}\right) e^{j(\pi/2 - 2\pi/3)}$$
$$= 2\left(\tfrac{-\sqrt{3}}{2}\right) e^{j(3\pi/6 - 4\pi/6)}$$
$$= -\sqrt{3}e^{-j\pi/6} = \sqrt{3}e^{j\pi}e^{-j\pi/6} = \sqrt{3}\, e^{j5\pi/6}$$

$\therefore$ OUTPUT IS: $\quad y[n] = \sqrt{3}\cos\left(\frac{\pi n}{3} + \frac{5\pi}{6}\right)$

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PROBLEM 7.14:

$$H(z) = 1 - 2z^{-2} - 4z^{-4}$$

$$h[n] = \delta[n] - 2\delta[n-2] - 4\delta[n-4]$$

$$x[n] = 20e^{j0n} + 20\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{4}\right) - 20\delta[n]$$

$$H(e^{j0})\cdot 20 \qquad \text{Need } H(e^{j\pi/2}) \qquad -20h[n]$$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j2\hat{\omega}} - 4e^{-j4\hat{\omega}}$$

$$H(e^{j0}) = 1 - 2 - 4 = -5$$

$$H(e^{j\pi/2}) = 1 - 2e^{-j\pi} - 4e^{-j2\pi}$$

$$= 1 + 2 - 4 = -1$$

$$\therefore y[n] = -100 - 20\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{4}\right) - 20\delta[n] + 40\delta[n-2] + 80\delta[n-4]$$

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PROBLEM 7.15:

$$x(t) = 4 + \cos(250\pi t - \pi/4) - 3\cos\left(\frac{2000\pi}{3} t\right)$$

with $f_s = 1000$

$$x[n] = x(t)\Big|_{t=n/f_s} = 4 + \cos\left(\frac{\pi n}{4} - \frac{\pi}{4}\right) - 3\cos\left(\frac{2\pi}{3} n\right).$$

Now, run $x[n]$ through the filter $H(z)$.
To do so, we need frequency response at $\hat{\omega} = 0,\ \pi/4,\ 2\pi/3$ $\quad H(e^{j\hat{\omega}}) = \dfrac{1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}}{3}$

$$H(e^{j0}) = \frac{1+1+1}{3} = 1$$

$$H(e^{j\pi/4}) = \frac{1}{3}\left(1 + e^{-j\pi/4} + e^{-j\pi/2}\right) = \frac{1}{3}\left(1 + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} - j\right)$$

$$= \frac{2+\sqrt{2}}{6} - j\,\frac{2+\sqrt{2}}{6} = 0.569 - j0.569 = .8047\,e^{-j\pi/4}$$

$$H(e^{j2\pi/3}) = \frac{1}{3}\left(1 + e^{-j2\pi/3} + e^{-j4\pi/3}\right) = 0$$

So, the output of the digital filter is:

$$y[n] = 4 + 0.8047\cos\left(\frac{\pi}{4}n - \frac{\pi}{2}\right) + 0$$

Now convert back to analog

$$n \longrightarrow f_s t = 1000t$$

$$y(t) = 4 + 0.8047\cos(250\pi t - \pi/2)$$

$$\text{or} \quad = 4 + 0.8047\sin(250\pi t)$$