PROBLEM 6.4:

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

(a) $$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{2} b_k e^{-j\hat{\omega}k} = 2 - 3e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}}$$

Simplify with symmetry:

$$\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}\left\{2e^{j\hat{\omega}} - 3 + 2e^{-j\hat{\omega}}\right\}$$

$$= \underbrace{(4\cos\hat{\omega} - 3)}_{\text{THIS TERM IS MAG, EXCEPT FOR A POSSIBLE MINUS SIGN}} \underbrace{e^{-j\hat{\omega}}}_{\text{PHASE}}$$

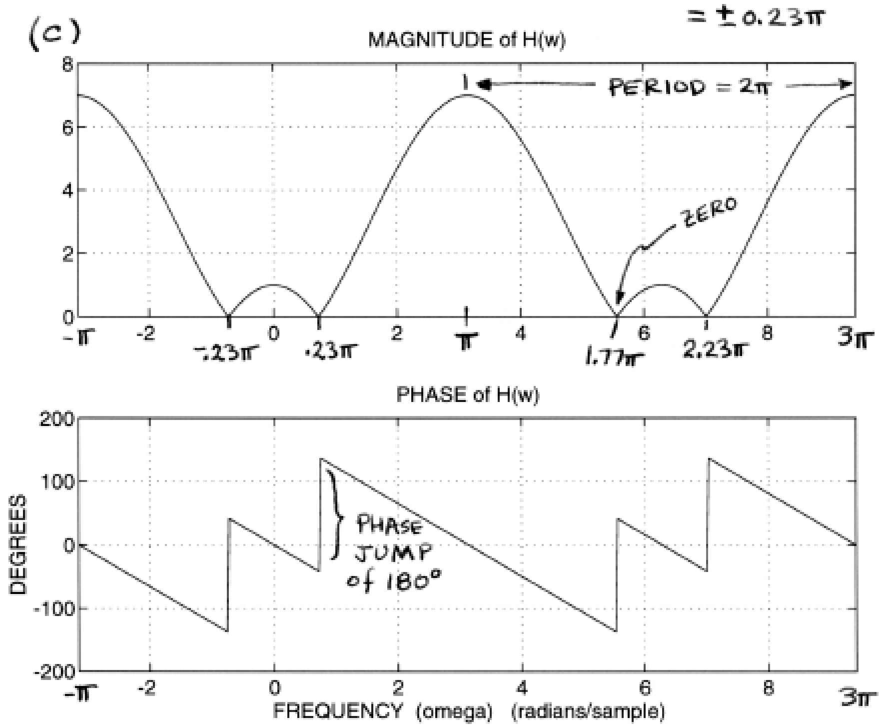
(b) $\cos\hat{\omega}$ has PERIOD $= 2\pi \Rightarrow \mathcal{H}(\hat{\omega})$ has period $= 2\pi$

(d) OUTPUT IS ZERO WHEN $\mathcal{H}(\hat{\omega}) = 0$

$\Rightarrow$ SOLVE $4\cos\hat{\omega} - 3 = 0 \Rightarrow \hat{\omega} = \cos^{-1}(3/4)$

$= \pm 0.23\pi$

(c)



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PROBLEM 6.4 (more):

(e) $\mathcal{H}(\hat{\omega})$ at $\hat{\omega}=\pi/13$ is $\mathcal{H}(\pi/13)=0.8838e^{-j0.077\pi}$

Since the freq. response alters mag & phase of the input, we can get output via:

$$x[n]=\sin(\tfrac{\pi}{13}n)=\cos(\tfrac{\pi}{13}n-\tfrac{\pi}{2}).$$

$$\Rightarrow y[n]=0.8838\cos(\tfrac{\pi}{13}n-0.5\pi-0.077\pi)$$

$$=0.8838\cos(\tfrac{\pi}{13}n-0.577\pi) \quad \underline{\text{for all } n}$$

Another approach which uses linearity:

$$\sin(\tfrac{\pi}{13}n)=\frac{1}{2j}e^{j\pi n/13}-\frac{1}{2j}e^{-j\pi n/13}$$

$$\Rightarrow y[n]=\mathcal{H}(\tfrac{\pi}{13})\frac{1}{2j}e^{j\pi n/13}-\mathcal{H}(-\tfrac{\pi}{13})\frac{1}{2j}e^{-j\pi n/13}$$

$$=\frac{1}{2j}\left\{0.8838e^{j(\pi n/13-0.077\pi)}-0.8838e^{-j(\pi n/13-0.077\pi)}\right\}$$

$$=0.8838\sin(\tfrac{\pi}{13}n-0.077\pi)$$

or $y[n]=0.8838\cos(\tfrac{\pi}{13}n-0.577\pi).$

NOTE: THIS METHODS TRACKS THE POSITIVE AND NEGATIVE FREQUENCY COMPONENTS THROUGH THE SYSTEM SEPARATELY.

PROBLEM 6.5:

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

(a) use filter coeffs: $\{b_k\} = \{1, 2, 1\}$

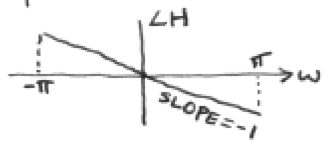
$$\mathcal{H}(\hat\omega) = 1 + 2e^{-j\hat\omega} + e^{-j2\hat\omega}$$

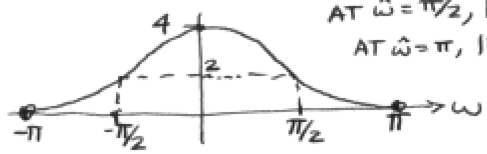
(b) $\mathcal{H}(\hat\omega) = e^{-j\hat\omega}\left(e^{j\hat\omega} + 2 + e^{-j\hat\omega}\right) = e^{-j\hat\omega}\left(2 + 2\cos\hat\omega\right)$

phase $= -\hat\omega$ MAG $= 2 + 2\cos\hat\omega$ $|H| = 4$ at $\hat\omega = 0$

AT $\hat\omega = \pi/2$, $|H| = 2$

AT $\hat\omega = \pi$, $|H| = 0$





(c) $x[n] = 10 + 4\cos\left(\frac{\pi}{2}n + \pi/4\right)$

$$= 10 + 2e^{j\pi/4}e^{j\pi/2\, n} + 2e^{-j\pi/4}e^{-j\pi/2\, n}$$

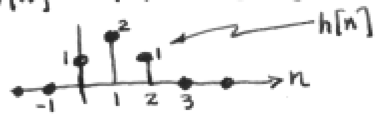
$$y[n] = 10\mathcal{H}(0) + \mathcal{H}(\pi/2)\,2e^{j\pi/4}e^{j\pi/2\, n} + 2\mathcal{H}(-\pi/2)\,e^{-j\pi/4}e^{-j\frac{\pi}{2}n}$$

$$\mathcal{H}(0) = 4e^{j0} \qquad \mathcal{H}(\pi/2) = e^{-j\pi/2}(2) \qquad \mathcal{H}(-\pi/2) = 2e^{j\pi/2}$$

$$\Rightarrow y[n] = 40 + 4e^{-j\pi/2}e^{j\pi/4}e^{j\pi/2\, n} + 4e^{j\pi/2}e^{-j\pi/4}e^{-j\pi/2\, n}$$

$$= 40 + 8\cos\left(\frac{\pi}{2}n - \pi/4\right)$$

(d) $x[n] = \delta[n] \implies y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$



(e) $x[n] = u[n]$

$y[n] = u[n] + 2u[n-1] + u[n-2]$

$y[n] = 0$ for $n < 0$

$y[0] = u[0] + 2u[-1] + u[-2] = 1 + 0 + 0 = 1$

$y[1] = u[1] + 2u[0] + u[-1] = 1 + 2 + 0 = 3$

$y[2] = u[2] + 2u[1] + u[0] = 1 + 2 + 1 = 4$

$y[n] = 4$ for $n \geq 2$.

PROBLEM 6.7:

(a) $H(e^{j\hat{\omega}}) = 1 + 2e^{-j3\hat{\omega}}$

Solution: Use the fact that the frequency response for $\delta[n - n_0]$ is $H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}$.

$$h[n] = \delta[n] + 2\delta[n-3]$$

(b) $H(e^{j\hat{\omega}}) = 2e^{-j3\hat{\omega}}\cos(\hat{\omega})$

Solution: Use the inverse Euler formula to write the frequency response in terms of complex exponentials.

$$H(e^{j\hat{\omega}}) = 2e^{-j3\hat{\omega}}\cos(\hat{\omega}) = e^{-j3\hat{\omega}}\left(e^{j\hat{\omega}} + e^{-j\hat{\omega}}\right)$$
$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$$
$$\Rightarrow h[n] = \delta[n-2] + \delta[n-4]$$

(c) $H(e^{j\hat{\omega}}) = e^{-j4.5\hat{\omega}}\dfrac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)}$

Solution: Use the fact that the frequency response for an L-point running sum filter is:

$$H_L(e^{j\hat{\omega}}) = e^{-j\hat{\omega}(L-1)/2}\frac{\sin(L\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

Thus, we see that $L/2 = 5$, or $L = 10$, and we can rewrite the frequency response as

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}(10-1)/2}\frac{\sin(10\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

because $(L-1)/2$ is equal to 4.5 when $L = 10$. Having made these identifications in the formula for $H(e^{j\hat{\omega}})$, we get the impulse response of the 10-point running-sum filter:

$$\begin{aligned} h[n] &= u[n] - u[n-10] \\ &= \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]\ldots \\ &\quad + \delta[n-5] + \delta[n-6] + \delta[n-7] + \delta[n-8] + \delta[n-9] \end{aligned}$$

PROBLEM 6.9:

(a)
$$\mathcal{H}(\hat{\omega}) = (1-e^{-j\hat{\omega}})\left(1 - 2(0.5)\cos\tfrac{\pi}{6}\, e^{-j\hat{\omega}} + (0.5)^2 e^{-j2\hat{\omega}}\right)$$
$$= (1-e^{-j\hat{\omega}})\left(1 - \tfrac{\sqrt{3}}{2} e^{-j\hat{\omega}} + \tfrac{1}{4} e^{-j2\hat{\omega}}\right)$$
$$= 1 \underbrace{- \tfrac{1}{2}(\sqrt{3}+2)}_{-1.866} e^{-j\hat{\omega}} + \underbrace{\left(\tfrac{1}{4} + \tfrac{\sqrt{3}}{2}\right)}_{1.116} e^{-j2\hat{\omega}} - \tfrac{1}{4} e^{-j3\hat{\omega}}$$

Difference Equation:
$$y[n] = x[n] - 1.866\,x[n-1] + 1.116\,x[n-2] - \tfrac{1}{4}\,x[n-3]$$

(b) when $x[n] = \delta[n]$, $y[n] = h[n]$ <u>impulse</u> <u>response</u>
$$h[n] = \delta[n] - 1.866\,\delta[n-1] + 1.116\,\delta[n-2] - \tfrac{1}{4}\,\delta[n-3]$$

(c) Find $\hat{\omega}$ where $\mathcal{H}(\hat{\omega}) = 0$

The only frequency is $\hat{\omega} = 0$, because then the factor $(1-e^{-j\hat{\omega}}) = 0$. The other two factors in $\mathcal{H}(\hat{\omega})$ are never zero for $-\pi \le \hat{\omega} \le \pi$.

PROBLEM 6.13:

(a) $y[n] = y_3[n] = x_3[n-1] + x_3[n-2]$ $\qquad x_3[n] = y_2[n]$

$$= y_2[n-1] + y_2[n-2]$$

$$= (x_2[n-1] + x_2[n-3]) + (x_2[n-2] + x_2[n-4])$$

Now replace $x_2[n]$ with $y_1[n]$

$$y[n] = y_1[n-1] + y_1[n-2] + y_1[n-3] + y_1[n-4]$$

$$= (x_1[n-1] - x_1[n-2]) + (x_1[n-2] - x_1[n-3])$$

$$+ (x_1[n-3] - x_1[n-4]) + (x_1[n-4] - x_1[n-5])$$

CANCEL

$$y[n] = x_1[n-1] - x_1[n-5] \qquad x_1[n] = x[n]$$

$$y[n] = x[n-1] - x[n-5]$$

(b) Same thing as part (a) but use $\mathcal{H}_i(\hat{\omega})$

$$\left.\begin{aligned} \mathcal{H}_1(\hat{\omega}) &= 1 - e^{-j\hat{\omega}} \\ \mathcal{H}_2(\hat{\omega}) &= 1 + e^{-j2\hat{\omega}} \\ \mathcal{H}_3(\hat{\omega}) &= e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \end{aligned}\right\} \text{Multiply these together}$$

$$\mathcal{H}(\hat{\omega}) = \mathcal{H}_1(\hat{\omega})\mathcal{H}_2(\hat{\omega})\mathcal{H}_3(\hat{\omega})$$

$$= (1 - e^{-j\hat{\omega}})(1 + e^{-j2\hat{\omega}})(e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$$

$$= (1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}})(e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$$

$$= e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} - e^{-j4\hat{\omega}}$$

$$+ e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$\Rightarrow \quad y[n] = x[n-1] - x[n-5]$$

PROBLEM 6.18:

$$x[n] = \underbrace{5}_{\text{Need } \mathcal{H}(0)} + \underbrace{20\cos(\tfrac{\pi}{2}n + \tfrac{\pi}{4})}_{\text{DEPENDS on } \mathcal{H}(\pi/2)} + \underbrace{10\delta[n-3]}_{\text{Need impulse response } h[n]}$$

$$\mathcal{H}(0) = (1-j)(1-(-j))(1+1)$$
$$= (1-j)(1+j)2 = 2\cdot 2 = 4$$

$$\mathcal{H}(\pi/2) = (1-je^{-j\pi/2})(1+je^{-j\pi/2})(1+e^{-j\pi/2})$$
$$= (1-j(-j))(1+j(-j))(1-j)$$
$$= (1-1)(1+1)(1-j) = 0$$

To find h[n], multiply out $\mathcal{H}(\hat{\omega})$

$$\mathcal{H}(\hat{\omega}) = (1 - je^{-j\hat{\omega}} + je^{-j\hat{\omega}} + e^{-j2\hat{\omega}})(1+e^{-j\hat{\omega}})$$
$$= (1+e^{-j2\hat{\omega}})(1+e^{-j\hat{\omega}})$$
$$= 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}}$$

$$\Rightarrow h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

Finally,

$$y[n] = 5(4) + 0 + 10h[n-3]$$
$$= 20 + 10\delta[n-3] + 10\delta[n-4] + 10\delta[n-5] + 10\delta[n-6]$$

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