PROBLEM 3.2:

(a) Read values from the graph:

$$x(t) = 4e^{j\pi/2} e^{-j2\pi(175)t} + 7e^{j\pi/3} e^{-j2\pi(50)t} + 11e^{j0t}$$
$$+ 4e^{-j\pi/2} e^{j2\pi(175)t} + 7e^{-j\pi/3} e^{+j2\pi 50t}$$

• Combine the positive & negative freqs:

$$x(t) = 8\cos(2\pi(175)t - \pi/2) + 14\cos(2\pi(50)t - \pi/3) + 11$$

(b) Cosine at $\omega = 2\pi(175)$ has period $= \frac{2\pi}{\omega} = \frac{1}{175}$

Cosine @ $\omega = 2\pi(50) \Rightarrow$ period $= \frac{1}{50}$

11 is a constant $\Rightarrow$ freq $= 0 \Rightarrow$ any period.

Need to find common period:

$\Rightarrow$ Solve $\ell_1\left(\frac{1}{50}\right) = \ell_2\left(\frac{1}{175}\right)$ $\quad \ell_1$ & ℓ_2 integers

$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{50}{175} = \frac{2}{7}$ REDUCED TO LOWEST TERMS

$\therefore$ min period $= \frac{2}{50} = \frac{1}{25}$ sec.

(c) $y(t) = A\cos(\omega_0 t + \varphi)$

$$= \Re e\{A e^{j\varphi} e^{j\omega_0 t}\}$$
$$= \tfrac{1}{2}Ae^{j\varphi}e^{j\omega_0 t} + \tfrac{1}{2}Ae^{-j\varphi}e^{-j\omega_0 t}$$

$Z_1 = \frac{1}{2}Ae^{j\varphi}$ $\qquad Z_1^* = \frac{1}{2}Ae^{-j\varphi}$ $\qquad$ NEGATIVE FREQ

PROBLEM 3.6:

$$x(t) = [12 + 7\sin(\pi t - \pi/3)]\cos 13\pi t$$

(a) $$x(t) = \left[12 + \frac{7}{2j}e^{j(\pi t - \pi/3)} - \frac{7}{2j}e^{-j(\pi t - \pi/3)}\right]\left(\frac{1}{2}e^{j13\pi t} + \frac{1}{2}e^{-j13\pi t}\right)$$

Multiply it all out:

$$x(t) = 6e^{j13\pi t} + 6e^{-j13\pi t} + \frac{7}{4}e^{-j\pi/2}e^{-j\pi/3}e^{j14\pi t} + \frac{7}{4}e^{-j5\pi/6}e^{-j12\pi t}$$
$$+ \frac{7}{4}e^{j\pi/2}e^{j\pi/3}e^{j12\pi t} + \frac{7}{4}e^{j5\pi/6}e^{-j14\pi t}$$

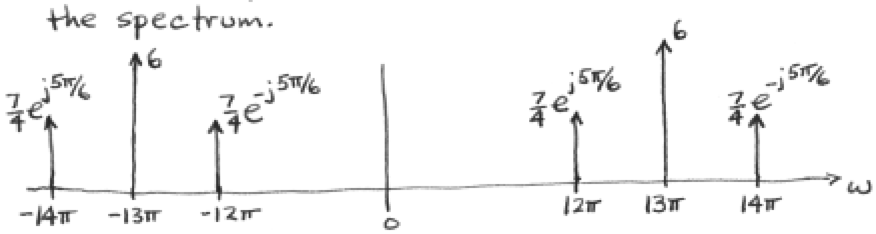
COMBINE

$$x(t) = 12\cos(13\pi t) + \frac{7}{2}\cos(14\pi t - 5\pi/6) + \frac{7}{2}\cos(12\pi t + 5\pi/6)$$

$\omega_1 = 12\pi$ rad/sec $\quad \omega_2 = 13\pi$ rad/s $\quad \omega_3 = 14\pi$ rad/s

$A_1 = 7/2 \quad \varphi_1 = 5\pi/6 \quad A_2 = 12 \quad \varphi_2 = 0 \quad A_3 = \frac{7}{2} \quad \varphi_3 = -\frac{5\pi}{6}$

(b) use the complex exponential form to sketch the spectrum.



SPECTRUM

PROBLEM 3.8:

$x(t)$ has four components:

$2 = 2e^{j0} \Rightarrow$ freq $= 0$, period = anything

$4\cos(40\pi t - \pi/5) = \Re e\{4e^{-j\pi/5}e^{j40\pi t}\}$ freq $= 20$Hz, period $= 1/20$ sec.

$3\sin(60\pi t) = 3\cos(60\pi t - \pi/2)$
$= \Re e\{3e^{-j\pi/2}e^{j60\pi t}\}$ freq $= 30$Hz, period $= 1/30$ sec.

$4\cos(120\pi t - \pi/3) = \Re e\{4e^{-j\pi/3}e^{j120\pi t}\}$ freq $= 60$ Hz, period $= 1/60$ sec.

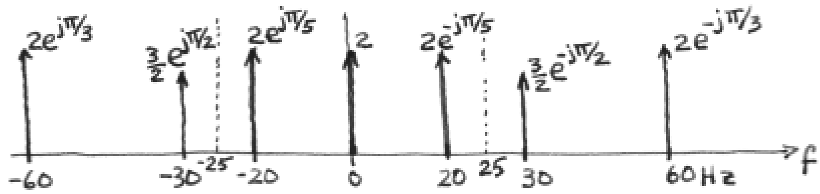
(a) The fundamental period is the smallest time that is exactly divisible by $1/20$, $1/30$ and $1/60$.

$\Rightarrow T_0 = 1/10$ sec because $T_0 = 2(1/20) = 3(1/30) = 6(1/60)$

$\Rightarrow f_0 = 10$ Hz $\Rightarrow \omega_0 = 2\pi f_0 = 20\pi$ rad/sec

$X_0 = 2$, $X_2 = 4e^{-j\pi/5}$, $X_3 = 3e^{-j\pi/2}$, $X_6 = 4e^{-j\pi/3}$

(b)



NOTE: since $\Re e\{X_i\} = \frac{1}{2}X_i + \frac{1}{2}X_i^*$ we use $\frac{1}{2}X_i$ in the plot.

(c) $y(t) = x(t) + 10\cos(50\pi t - \pi/6)$

equals $\Re e\{10e^{-j\pi/6}e^{j50\pi t}\}$ freq $= 25$Hz, period $= 1/25$ sec

The new period must be divisible by $1/10$ & $1/25$

$\Rightarrow T_0 = 1/5$ because $T_0 = 5(1/25)$ and $2(1/10)$

$\Rightarrow f_0 = 5$Hz $\Rightarrow \omega_0 = 2\pi f_0 = 10\pi$ rad/sec

We must re-index the X_i because ω_0 is lower.

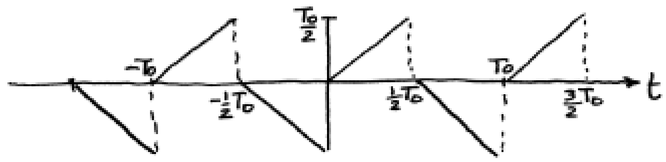
$Y_0 = 2$, $Y_4 = X_2$, $Y_5 = 10e^{-j\pi/6}$, $Y_6 = X_3$, $Y_{12} = X_6$

There will be one new pair of lines at $f = \pm 25$Hz

PROBLEM 3.10:

Half-wave symmetry: $x(t+T_0/2) = -x(t)$

(a) $x(t) = t$ for $0 \le t < T_0/2$



(b)
$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j0}\, dt$$

Change of variables $u = t - T_0/2$

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)\,dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} x(t)\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)\,dt + \frac{1}{T_0}\int_0^{T_0/2} \underbrace{x(u+T_0/2)}_{=-x(u)}\,du$$

$= -x(u)$ because of half-wave symmetry

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)\,dt - \frac{1}{T_0}\int_0^{T_0/2} x(u)\,du = \underline{\underline{0}}$$

(c) If k is even, then $k = 2\ell$

$$a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t)\, e^{-j(2\pi/T_0)2\ell t}\,dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} x(t)\, e^{-j(2\pi/T_0)2\ell t}\,dt$$

Make the same change of variables: $u = t - T_0/2$.

$$a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t)\, e^{-j(2\pi/T_0)2\ell t}\,dt + \frac{1}{T_0}\int_0^{T_0/2} \underbrace{x(u+T_0/2)}_{=-x(u)}\, e^{-j(2\pi/T_0)2\ell(u+T_0/2)}\,du$$

$$e^{-j(2\pi/T_0)2\ell u}\, e^{-j(2\pi/T_0)2\ell T_0/2} = e^{-j(2\pi/T_0)2\ell u}\,\underbrace{e^{-j2\pi\ell}}_{=1}$$

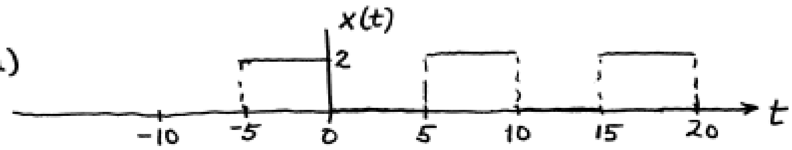
$$\Rightarrow a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t)\, e^{-j(2\pi/T_0)2\ell t}\,dt - \frac{1}{T_0}\int_0^{T_0/2} x(u)\, e^{-j(2\pi/T_0)2\ell u}\,du$$

$\Rightarrow a_k = 0$ for k even.

PROBLEM 3.12:

$$x(t) = \begin{cases} 0 & 0 \le t \le 5 \\ 2 & 5 < t \le 10 \end{cases} \qquad T_0 = 10 \text{ secs.}$$

(a)



(b) $a_0 = \frac{1}{T_0} \times \text{Area} = \frac{1}{10} \times (5 \times 2) = 1$

(c) $a_1 = \frac{1}{10} \int_5^{10} 2 e^{-j(2\pi/10)t} \, dt$

$$= \frac{1}{5} \left. \frac{e^{-j\pi t/5}}{(-j\pi/5)} \right|_5^{10} = \frac{e^{-j2\pi} - e^{-j\pi}}{-j\pi} = \frac{1-(-1)}{-j\pi} = \frac{2j}{\pi}$$

(d) $y(t) = 1 + x(t) = 1 + \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}$

$$= (1 + a_0) + \sum_{k \neq 0} a_k e^{j\omega_0 k t}$$

$$\Rightarrow b_0 = 1 + a_0 \quad \text{and} \quad b_1 = a_1$$

PROBLEM 3.14:

(a) $y(t) = Ax(t) = A \cdot \sum_k a_k e^{jk\omega_0 t}$

$$= \sum_k (Aa_k) e^{jk\omega_0 t} \quad \Longrightarrow \quad \underline{b_k = Aa_k}$$

(b) $y(t) = x(t - t_d)$

$$= \sum_k a_k e^{j\omega_0 k(t - t_d)}$$

$$= \sum_k \left(a_k e^{-j\omega_0 k t_d}\right) e^{j\omega_0 k t}$$

$$\Longrightarrow \quad \underline{b_k = a_k e^{-jk\omega_0 t_d}}$$