**PROBLEM 2.1:**

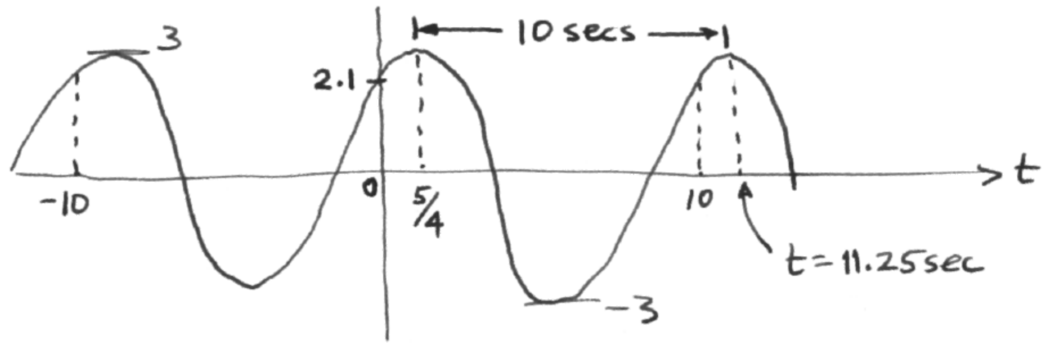
$$x(t) = 3\cos\left(\frac{\pi}{5}t - \frac{\pi}{4}\right)$$

$$2\pi f = \frac{\pi}{5} \Rightarrow f = \frac{1}{10} \Rightarrow T = 10 \text{ sec. (PERIOD)}$$

$$\varphi = -2\pi\frac{t_1}{T} \Rightarrow t_1 = -\frac{\varphi}{2\pi}T = \frac{\pi/4}{2\pi} \times 10 = \frac{5}{4} = 1.25\text{sec}$$

at $t=0$

$$x(t) = 3\cos(-\pi/4) = \frac{3\sqrt{2}}{2} \approx 2.1$$



**PROBLEM 2.5:**

(a) $\cos(\theta_1+\theta_2) = \Re e\{e^{j(\theta_1+\theta_2)}\} = \Re e\{e^{j\theta_1}e^{j\theta_2}\}$

$= \Re e\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)\}$

$= \Re e\{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + j(\text{other terms})\}$

$= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$

(b) $\cos(\theta_1-\theta_2) = \Re e\{e^{j(\theta_1-\theta_2)}\} = \Re e\{e^{j\theta_1}e^{-j\theta_2}\}$

$= \Re e\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 - j\sin\theta_2)\}$

$= \Re e\{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + j(\text{other terms})\}$

$= \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$

## PROBLEM 2.7:

(a) $3e^{j\pi/3} + 4e^{-j\pi/6} = \left(\frac{3}{2} + j\frac{3\sqrt{3}}{2}\right) + \left(4\frac{\sqrt{3}}{2} - j\frac{4}{2}\right)$

$= 4.9641 + j0.5981$

$= 5e^{j0.12}$ NOTE: $0.12$ rad $= 6.87°$

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(b) $\sqrt{3} - j3 = \sqrt{3+3^2}\, e^{-j\pi/3} = \sqrt{12}\, e^{-j\pi/3}$.

$\Rightarrow \left(\sqrt{3} - j3\right)^{10} = \left(\sqrt{12}\, e^{-j\pi/3}\right)^{10}$

$= 2^{10} 3^5 e^{-j10\pi/3}$ $\quad -\frac{10\pi}{3} + 4\pi = \frac{-10\pi + 12\pi}{3} = 2\pi/3$

$= 248{,}832\, e^{+j2\pi/3} = -124{,}416 + j215{,}494.83$

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(c) $\frac{1}{\sqrt{3} - j3} = \frac{1}{\sqrt{12}\, e^{-j\pi/3}} = \frac{1}{\sqrt{12}} e^{+j\pi/3} = 0.2887 e^{+j\pi/3}$

$= 0.14434 + j0.25$

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(d) $\left(\sqrt{3} - j3\right)^{1/3} = \left(\sqrt{12}\, e^{-j\pi/3}\right)^{1/3} = \left(\sqrt{12}\, e^{-j(\pi/3 + 2\pi\ell)}\right)^{1/3}$

$= 12^{1/6} e^{-j(\pi/9 + \frac{2\pi}{3}\ell)}$ $\quad \ell =$ integer

Need $\ell = 0, 1, 2$

There are 3 answers:

$1.513 e^{-j\pi/9} = 1.422 - j0.5175$

$1.513 e^{-j7\pi/9} = -1.159 - j0.9726$

$1.513 e^{-j13\pi/9} = 1.513 e^{+j5\pi/9} = -0.2627 + j1.49$

---

(e) $\Re e\{ j e^{-j\pi/3} \} = \Re e\{ e^{j\pi/2} e^{-j\pi/3} \}$.

$= \Re e\{ e^{j\pi/6} \} = \cos(\pi/6) = \frac{\sqrt{3}}{2} = 0.866$

**PROBLEM 2.13:**

(a) $s_i(t) = \Im m\left\{ 5e^{j\pi/3} e^{j10\pi t} \right\} = \Im m\left\{ 5e^{j(10\pi t + \pi/3)} \right\}$

$$\Rightarrow \quad s_i(t) = 5\sin(10\pi t + \pi/3)$$

We can convert to the cosine form: $\sin\theta = \cos(\theta - \pi/2)$

$$s_i(t) = 5\cos(10\pi t - \pi/6)$$

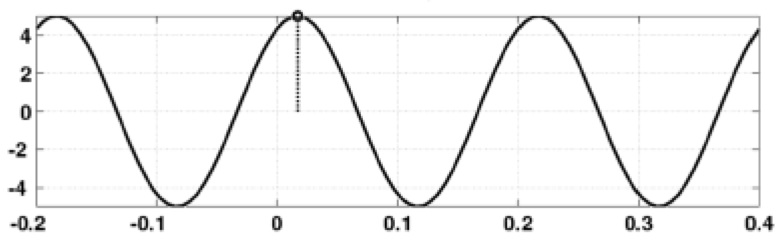
The period of $s_i(t)$ is $T = 1/5$, because $\frac{2\pi}{T} = 10\pi$

The value at $t=0$ is $s_i(0) = 5\cos(-\pi/6) = \frac{5\sqrt{3}}{2} = 4.33$

The peak is at $t = t_1$ where

$$10\pi t_1 - \pi/6 = 0 \quad \Rightarrow \quad t_1 = 1/60$$

Plot of $s_i(t)$



## PROBLEM 2.15:

Express $x(t) = 5\cos(\omega t + \frac{1}{3}\pi) + 7\cos(\omega t - \frac{5}{4}\pi) + 3\cos(\omega t)$ in the form $x(t) = A\cos(\omega t + \phi)$.

*Solution:*
Convert to phasors:

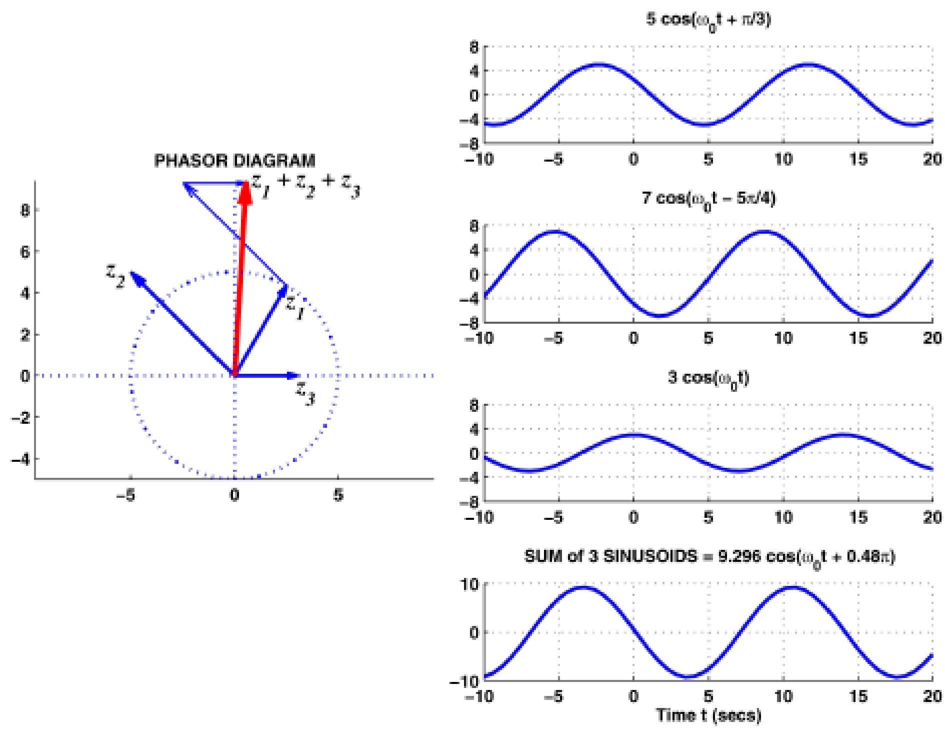
$$\begin{aligned} 5\cos(\omega t + \tfrac{1}{3}\pi) &\longrightarrow z_1 = 5e^{j\pi/3} = 2.5 + j4.33 \\ 7\cos(\omega t - \tfrac{5}{4}\pi) &\longrightarrow z_2 = 7e^{j5\pi/34} = -4.95 + j4.95 \\ 3\cos(\omega t) &\longrightarrow z_3 = 3e^{j0} = 3 + j0 \end{aligned}$$

Perform the phasor addition to get:

$$z_1 + z_2 + z_3 = (2.5 + j4.33) + (-4.95 + j4.95) + (3) = 0.5503 + j9.28 = 9.296e^{j0.48\pi}$$

Thus, the resultant sinusoid is:

$$x(t) = 9.296\cos(\omega t + 0.48\pi)$$



## PROBLEM 2.17:

$$x(t) = 5\cos(\omega_0 t + 3\pi/2) + 4\cos(\omega_0 t + 2\pi/3) + 4\cos(\omega_0 t + \pi/3)$$

(a) Express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of $A$ and $\phi$.

$$\left.\begin{aligned} z_1 &= 5e^{j3\pi/2} = 0 - 5j \\ z_2 &= 4e^{j2\pi/3} = -2 + j3.46 \\ z_3 &= 4e^{j\pi/3} = 2 + j3.46 \end{aligned}\right\} \quad \begin{aligned} z &= z_1 + z_2 + z_3 \\ &= 0 + j1.928 \\ &= 1.928\, e^{j\pi/2} \end{aligned}$$

$$\therefore \quad x(t) = 1.928 \cos(\omega_0 t + \pi/2)$$

***

| Z = X | + jY | Magnitude | Phase | Ph/pi | Ph(deg) |
|---|---|---|---|---|---|
| -9.185e-16 | -5 | 5 | -1.571 | -0.500 | -90.00 |
| -2 | 3.464 | 4 | 2.094 | 0.667 | 120.00 |
| 2 | 3.464 | 4 | 1.047 | 0.333 | 60.00 |
| 4.441e-16 | 1.928 | 1.928 | 1.571 | 0.500 | 90.00 |

(b) Plot all the phasors used to solve the problem in part (a) in the complex plane.