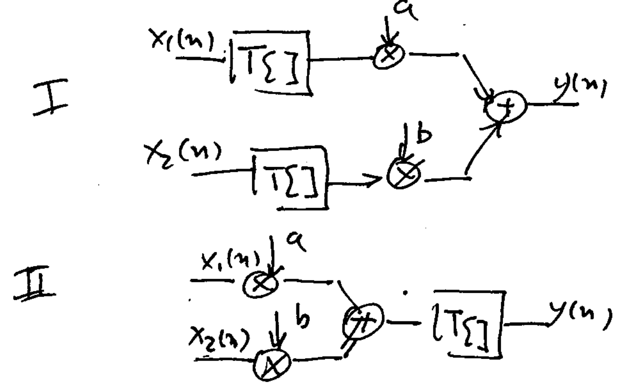
Problem 1 (15 points)

Determine whether or not the system described by the difference equation

$$y(n) = \sin(0.11\pi n)x(n) + x(n+1)$$

is linear, time invariance and causal. Prove your statements using the definitions.

LINEARITY



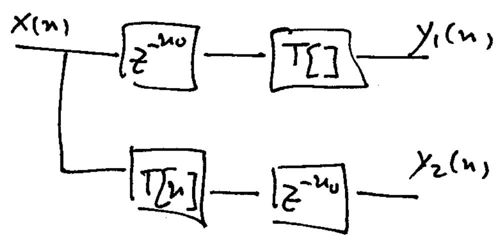
$$y_1(n) = a[\sin(0.11\pi n)x_1(n) + x_1(n+1)]$$

$$y_2(n) = b[\sin(0.11\pi n)x_1(n) + x(n+1)]$$

$$y(n) = \sin(0.11\pi n)\left(a x_1(n) + b x_2(n)\right) + \left(a x_1(n+1) + b x_2(n+1)\right)$$

this is EXACTLY THE OUTPUT OF BLOCK II, SO SYSTEM IS LINEAR

TIME INVARIANCE



$$y_1(n) = \sin(0.11\pi n)\, x(n-n_0) + x(n+1-n_0)$$

$$y_2(n) = \{\sin(0.11\pi n)\, x(n) + x(n+1)\} z^{-n_0} =$$

$$= \sin(0.11\pi(n-n_0))\, x(n-n_0) + x(n+1-n_0)$$

THEY ARE DIFFERENT HENCE SYSTEM IS NOT TIME INVARIANT

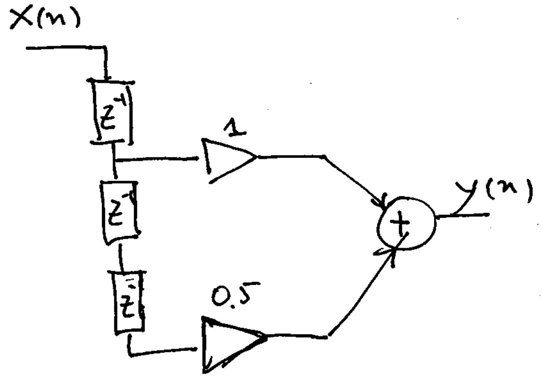
Problem 2 (15 points)

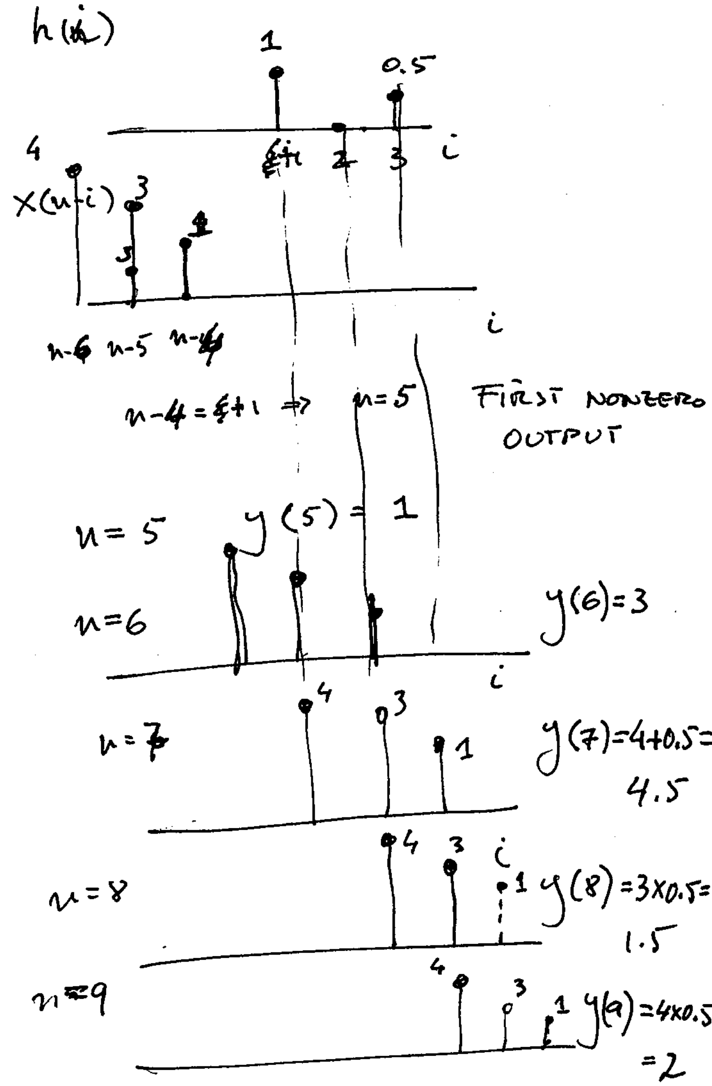
A LTI system has an impulse response

$$h(n) = \delta(n-1) + 0.5\delta(n-3)$$

Draw the block diagram for this LTI system. Compute the output using convolution when the input is $x(n) = \delta(n-4) + 3\delta(n-5) + 4\delta(n-6)$.

OR GRAPHICAL METHOD





$y(n) = 0 \quad n \leq 4$

$y(n) = 1 \quad n = 5$

$y(n) = 3 \quad n = 6$

$y(n) = 4 + 0.5 = 4.5 \quad n = 7$

$y(n) = 3 \times 0.5 = 1.5 \quad n = 8$

$y(n) = 4 \times 0.5 = 2 \quad n = 9$

$y(n) = 0 \quad n \geq 10$

$y(n) = 0 \quad n \geq 10$

Problem 3 (15 points)

A FIR is described by the difference equation

$$y(n) = x(n) + 2x(n-1) + 3x(n-2)$$

(5) 3a) Determine the frequency response of the system

(5) 3b) Sketch the magnitude and ~~the phase responses~~ ← WILL NOT GRADE THE PHASE PLOT !

(5) 3c) Determine the response of this system to $x(n) = 10 + 4\cos(0.5\pi n + \pi/4)$

3a) $h(n) = \delta(n) + 2\,\delta(n-1) + 3\,\delta(n-2)$

So $H(e^{j\hat{w}}) = e^{j0} + 2\,e^{-j\hat{w}} + 3\,e^{-j2\hat{w}} = 1 + 2e^{-j\hat{w}} + 3e^{-j2\hat{w}}$

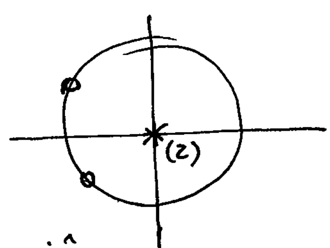
3b) $H(z) = 1 + 2z^{-1} + 3z^{-2} = \dfrac{z^2 + 2z + 3}{z^2}$

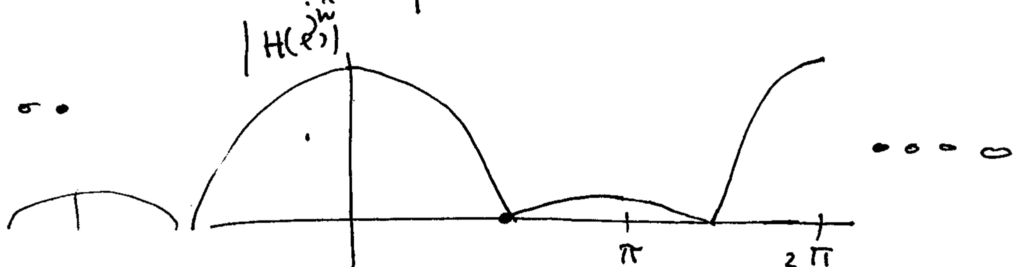
Roots $= \dfrac{-2 \pm \sqrt{4-12}}{2} = -1 \pm j\dfrac{\sqrt{8}}{2}$

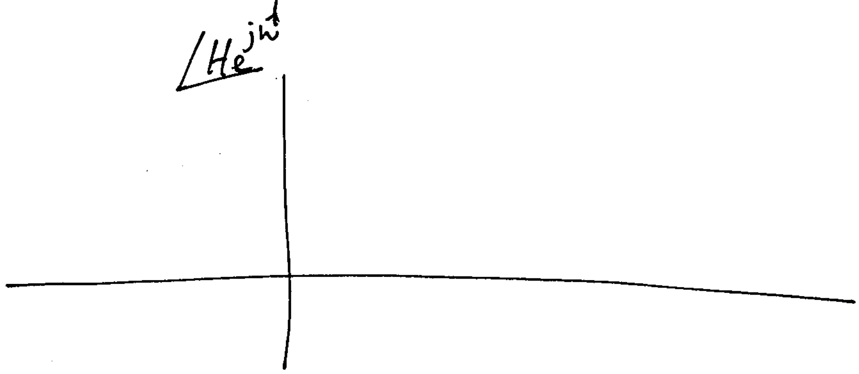
$r = \sqrt{1 + \frac{8}{4}} = \sqrt{3}$

$\theta \approx 150^\circ$

POLE ZERO PLOT







WILL NOT GRADE THE PHASE PLOT !

$$H(e^{j\hat{w}}) = 1 + 2e^{-j\hat{w}} + 3e^{-2jw}$$

$$x(n) = 10 + 4\cos\left(0.5\pi n + \frac{\pi}{4}\right) = x_1(n) + x_2(n)$$

OUTPUT OF $x_1(n)$

$$w = 0 \implies H(e^{j\hat{0}}) = 1 + 2 + 3 = 6$$

$$\text{So} \quad y_1(n) = 60$$

OUTPUT OF $x_2(n)$

$$w = \frac{\pi}{2} \quad \text{So} \quad H(e^{j\frac{\pi}{2}}) = 1 + 2e^{-j\frac{\pi}{2}} + 3e^{-j\pi} = 1 - 2j - 3 = -(2+2j)$$

$$= -2(1+j) = -2\sqrt{2}e^{j\frac{\pi}{4}}$$

$$\begin{cases} r = \sqrt{2} \\ \theta = 45^{\circ} \end{cases}$$

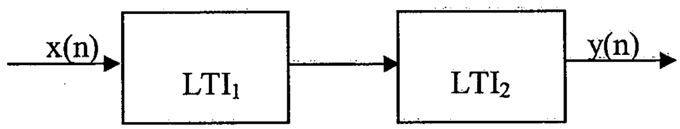
$$\text{So} \quad y_2(n) = -8\sqrt{2}\cos\left(0.5\pi n + \frac{\pi}{2}\right)$$

and the total output is

$$y(n) = 60 - 8\sqrt{2}\cos\left(0.5\pi n + \frac{\pi}{2}\right)$$

Problem 4 (15 points)

For the cascade system



where $H_1(z) = 1 - z^{-2}$
$h_2(n) = 2\delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)$

(10) 4a) Determine the difference equation that relates y(n) to x(n).
(2) 4b) If you flip the order of the two blocks does the difference equation change? Justify your answer.
(3) 4c) If you use finite dynamic range arithmetic (i.e. a preset of bits to represent the results of the calculations) the response of the cascade system to arbitrary inputs is NOT invariant to the ordering of the blocks because of saturation in the arithmetic calculations in one of the blocks. By observing the filter types ($H_1(z)$ and $H_2(z)$) construct an input signal that will demonstrate this statement by showing saturation in one of the orders and not in the other.

4a) $H_2(z) = 2 + z^{-1} + z^{-2} + z^{-3}$

So $H(z) = H_1(z)\, H_2(z) = (1-z^{-2})(2+z^{-1}+z^{-2}+z^{-3}) =$

$= 2 + z^{-1} + z^{-2} + z^{-3} - 2z^{-2} - z^{-3} - z^{-4} - z^{-6} =$

$= 2 + z^{-1} - z^{-2} - z^{-4} - z^{-6}$

HENCE $y(n) = 2x(n) + x(n-1) - x(n-2) - x(n-4) - x(n-6)$

4b) THE ANSWER DOES NOT CHANGE DUE TO THE COMMUTATIVE PROPERTY OF Z TRANSFORM ($H(z) = H_1(z).H_2(z) = H_2(z)\,H_1(z)$)

4c) $H_1(z)$ IS A BANDPASS FILTER WITH ZERO GAIN AT ZERO.
$H_2(z)$ IS A LOWPASS FILTER WITH A GAIN OF 5 AT ZERO

SO IF I CHOOSE A STEP FUNCTION AS MY INPUT WHEN $H_1(z).H_2(z)$ THE D.C. TERM WILL DISAPPEAR AT THE OUTPUT OF $H_1(z)$, AND SO WILL NOT BE A PROBLEM FOR THE LOWPASS FILTER.

NOW IF A STEP FUNCTION IS APPLIED TO $H_2(z) \cdot H_1(z)$, THE LOWPASS FILTER WILL SMOOTH + AMPLIFY THE STEP FUNCTION, CREATING POSSIBLY A SATURATION (OVERFLOW) IN THE COMPUTATION.

IN THIS CASE THE OUTPUT WILL BE DISTORTED AND WILL NOT BE EQUAL TO ~~THE~~ OUTPUT OF $H_1(z) \cdot H_2(z)$

Problem 5 (20 points)

For the 8[th] point (L=8) running sum averager $H(z)=\sum_{k=0}^{7} z^{-k}$

(5) 5a) Determine a close form solution for its transfer function.

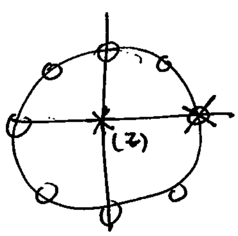
(5) 5b) Show its pole-zero plot and interpret it as a pole-zero cancellation of two filters that you need to specify the transfer functions.

(5) 5c) Sketch the magnitude of the frequency response H(z) labeling the frequency nulls and showing the magnitude value at zero frequency.

(5) 5d) Using the result of 5b), write the transfer function of the bandpass filter with center frequency at $\pi/4$ and which has real coefficients.

5a) $$H(z) = \sum_{k=0}^{7} z^{-k} = \frac{1-z^{-8}}{1-z^{-1}} = \frac{z^{8}-1}{z^{7}(z-1)}$$

5b)



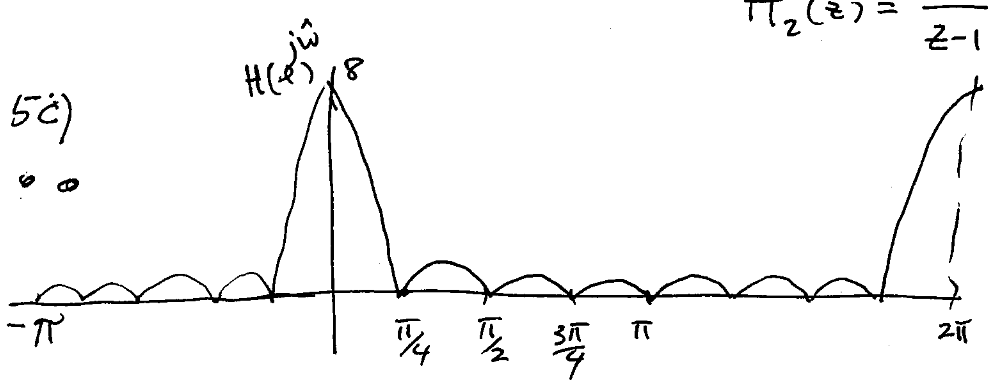
THIS CAN BE INTERPRETED AS A COMBFILTER

$$H_1(z) = \frac{z^8-1}{z^8}$$

CASCADED WITH A 1st ORDER IIR

$$H_2(z) = \frac{z}{z-1}$$

5c)



$H(z)|_{z=0} = 8$

5d) CAN USE THE COMB FILTER $H_1(z) = 1-z^{-8}$ AND CANCEL THE ZEROS AT $z = e^{j\pi/4}, e^{-j\pi/4}$ WITH A PAIR OF POLES AT SAME FREQUENCIES.

$$H_3(z) = (z - e^{-j\pi/4})(z - e^{j\pi/4}) =$$
$$z^2 - z(e^{j\pi/4} + e^{-j\pi/4}) + 1 = z^2 - 2z\cos\frac{\pi}{4} + 1 =$$
$$= 1 - \sqrt{2}z^{-1} + z^{-2}$$

So $$H(z) = \frac{(1-z^{-8})}{(1-\sqrt{2}z^{-1}+z^{-2})}$$

Problem 6 (20 points)

A continuous signal s(t) has a spectrum between 0-150Hz. During data collection, a sinusoidal interference (v(t)) with frequency 250Hz gets added to the signal to create the signal x(t)=s(t)+v(t). The signal x(t) is digitized to be further analyzed.

(3) 6a) Select a sampling frequency that obeys the Nyquist theorem for x(t) and show the spectrum of the digital signal x(n).

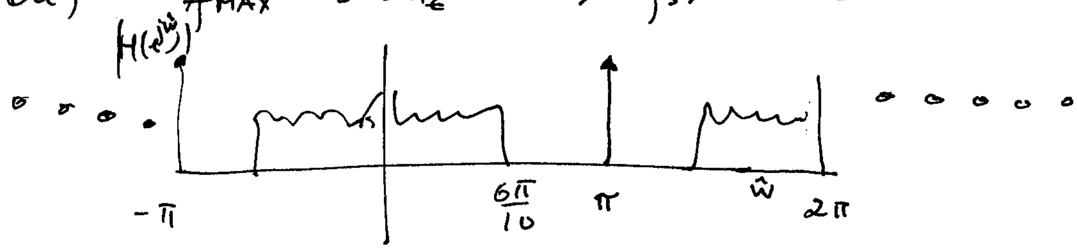
(7) 6b). Design a digital FIR filter to attenuate the noise interference v(n) and leave s(n) undisturbed. Note: there are many possible FIR filters, so choose a simple one.

(7) 6c) Another alternative is to sample x(t) at the Nyquist frequency of s(t). First specify this sampling frequency. Then find the digital frequency of the interference for this new sampling frequency. Finally, design a bandstop filter to null the interference v(n) and specify its transfer function.

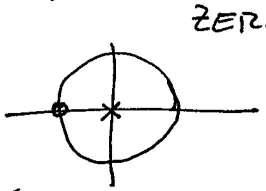
(3) 6d) Compare critically the solutions outlined in 6a&b and 6c in terms of preserving the structure of s(t) and also in terms of computation complexity (number of operations and number of samples created).

$\hat{\omega}_0 = \frac{2\pi \times 150}{500} = \frac{6\pi}{10}$

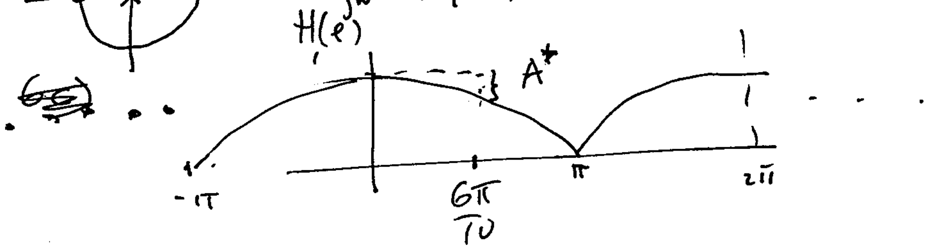
6a) $f_{MAX} = 250\,Hz \implies f_s \geq 500\,Hz$



6b) TO ATTENUATE THE SINUSOIDAL INTERFERENCE, PLACE A ZERO ~~WHEF~~ AT THE FREQ. IT OCCURS. IN THIS CASE IT IS AT π



$H_1(z) = 1 - e^{j\pi} z^{-1} = 1 + z^{-1}$



A* THERE WILL BE A SLIGHT ATTENUATION AT HIGH FREQ OF s(n)

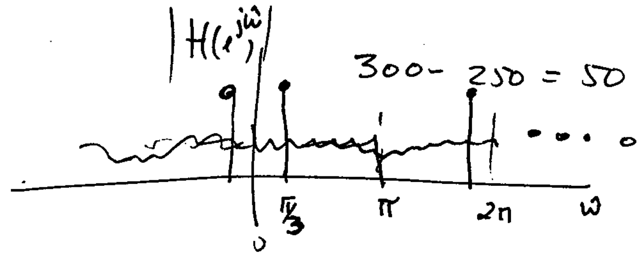
6 c)

$s(n) \longrightarrow 0,\ 150\,\text{Hz}$

So $f_{MAX} = 150\,\text{Hz} \Longrightarrow f_s \geq 300\,\text{Hz}$, PICK 300 Hz

$\hat{w}_0 = \frac{2\pi \times 250}{300} = \frac{2\pi\, 25}{30} > \pi$ SO NEED TO FIND ALIASED FREQ

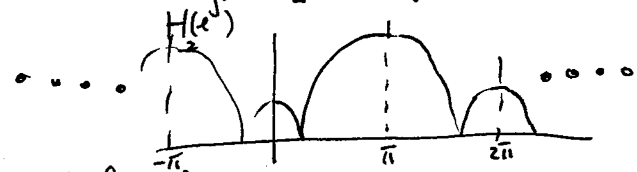
$300 - 250 = 50\,\text{Hz} \Longrightarrow \frac{2\pi\, 50}{300} = \frac{\pi}{3}$

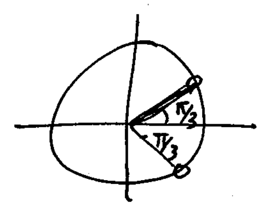


TO CANCEL THE SINUSOIDAL INTERFERENCE PUT A ZERO AT $\frac{\pi}{3}$

AND AT $-\frac{\pi}{3}$ FOR REAL COEFFICIENTS

$H_2(z) = (z - e^{j\pi/3})(z - e^{-j\pi/3})$





6 d) Solution 6a) WASTES BANDWIDTH, SINCE WE DO NOT NEED TO REPRESENT THE NOISE WITHOUT ALIASING (WE ARE GOING TO FILTER IT ANYWAY)

THE PROBLEM IS THAT THE NOISE FALLS BACK IN THE MIDDLE OF THE SIGNAL BAND, SO FILTERING BECOMES MORE DIFFICULT + WILL PROVIDE A LOCAL DISTORTION