HW#1

EEL 6825 – Fall 2001

Due Friday, September 7, 2001 in class. Late homework will lose $e^{\#\ of\ days\ late} - 1$ percentage points. Click on http://www.cnel.ufl.edu/hybrid/harris/latepoints.html to see the current penalty. A computer is not necessary for this assignment.

1. You are given the following two 1-D distributions which are valid for all values of x:

$$p(x|\omega_1) = \frac{1}{2}e^{-|x|}$$

$$p(x|\omega_2) = e^{-2|x|}$$

Assume that $P(\omega_1) = 1/3$ and $P(\omega_2) = 2/3$

   (a) Compute the posterior probability $P(\omega_i|x)$ for each class.

   (b) Derive the Bayes classifier for this problem. In other words, how would you classify new data points x?

   (c) Sketch a plot that graphically indicates the Bayes error.

   (d) Compute the numerical value of the Bayes error for this problem.

   (e) Compute the value of the Bhatacharrya bound for this problem. Remember that these are not normal distributions.

2. Three one-dimensional distributions are given as uniform in [0,1] for ω_1, uniform in [0,2] for ω_2 and uniform in [0,3] for ω_3. Assume the a priori probabilities are equal.

   (a) Compute $P(\omega_i|x)$ for each class and sketch each function on a separate plot.

   (b) Describe the Bayes classifier for the three distributions. Be sure to describe the class for each possible value of x.

   (c) Compute the Bayes error for this problem.

3. Two normal distributions are characterized by:

$$P(\omega_1) = P(\omega_2) = 0.5$$

$$\mu_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mu_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Derive the analytic form and sketch the Bayes decision boundary for the following cases: (Also sketch some equi-probability contours for each distribution.)

(a)

$$\Sigma_1 = \Sigma_2 = I$$

(b)

$$\Sigma_1 = I$$

$$\Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

(c)

$$\Sigma_1 = \begin{bmatrix} 1 & .5 \\ .5 & 1 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

4. Problem 2-6 in DH&S

5. Problem 2-13 in DH&S

6. (Extra credit) 2-32 in DH&S