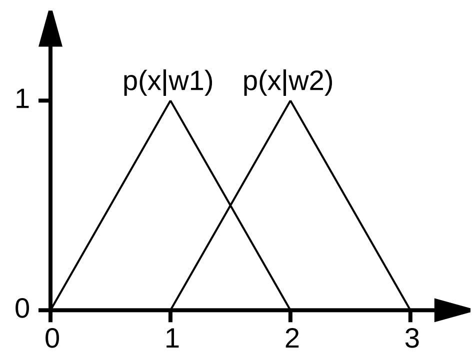
EEL6825 – Fall 2000
Exam 1 October 2, 2000

NAME: ______________________________

This exam is open-book and calculator. You may use any books or papers that you like. There are four problems in this exam, you have two full class periods. State your assumptions and reasoning for each problem. Justify your steps and clearly indicate your final answers.

1	
2	
3	
4	
TOTAL	

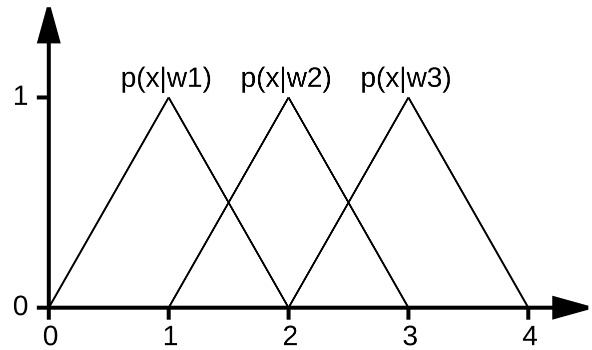
1. (25 points) You are given the following two 1-D distributions which are shown below. Assume that $P(\omega_1) = P(\omega_2)$. Answer the following questions



   (a) (5 points) Derive the Bayes classifier for this two-class problem. In other words, how would you classify new data points x?

(b) (10 points) Sketch a graph that indicates the Bayes error for the two-class problem. Compute the numerical value of the Bayes error for this problem.

(c) (10 points) Suppose we add a third distribution $p(x|\omega_3)$ as shown below. Assume that $P(\omega_1) = P(\omega_2) = P(\omega_3)$ Does the Bayes error for this three-class problem increase or decrease (compared to the 2-class problem)? Justify your answer.



2. (25 points) You are given two normal distributions with the following means and covariance matrices:

$$\mu_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \mu_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix}$$

Assume that $P(\omega_1) = e/(e+1) \approx .7311$ and $P(\omega_2) \approx 1/(e+1) = .2689$

(a) (5 points) Sketch one equal probability contour for each distribution. (Make sure that the contour you draw for each distribution specifies the same probability value). Clearly label your axes.

(b) (10 points) Derive the analytic form for the Bayes decision boundary.

(c) (5 points) Sketch the Bayes decision boundary on a plot that also shows the equiprobability contours. Clearly label your axes.

(d) (5 points) Compute the numerical value of the Bhatacharrya bound.

3. (25 points) Consider the following 1-D probability density function:

$$p(x) = \frac{1}{2}e^{-|x-\mu|}$$

Answer the following questions making any assumptions that you feel appropriate.

(a) (5 points) Sketch this probability distribution. Verify that this is a proper probability distribution function.

(b) (10 points) Derive the maximum likelihood estimate for μ given N data points $x_1, x_2, \ldots, x_N$.

(c) (10 points) Three points are sampled from this distribution–their values just so happen to be $x^{(1)} = 1$, $x^{(2)} = 2$ and $x^{(3)} = 17$. What is the value of μ for the most likely distribution to have generated these two points? (Note: you may be able to figure out this part even if you can't fully finish part b)

4. (25 points) Short Answer.

(a) (5 points) A certain 2-class linear classifier ($g(\underline{x}) = \underline{w}^T \underline{x} + w_0$) gives an error of 57% on some test data. Explain an extremely simple method to improve the performance of this classifier on the same test data.

(b) (5 points) In completing an assignment, a student generated 100 samples from two known Normal distributions and designed a Bayes classifier using the known parameter values. She was surprised to discover that the classification error on the samples was larger than the Bhattacharyya bound she computed from the known distribution parameters! Since the Bhattacharyya bound is supposed to be an upper bound on the Bayes error, can you explain her results?

(c) (5 points) Suppose that the probability density functions (and parameters) for two classes are identical. Describe how you would classify between the two classes and what the expected error would be.

(d) (5 points) Consider a two-class classification problem with

$$P(\omega_1) = P(\omega_2) = 0.5$$

Assume that data are sampled from Normal distributions. Make any other assumptions that you feel are necessary. Class ω_1 sample points are given by:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Class ω_2 sample points are given by:

$$\begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Compute the sampled mean and sampled covariance matrix for each class.

(e) (5 points) Derive the $\underline{w}$ vector given by the Fisher discriminant for the data in problem 4(d) on the last page. Make sure that you normalize the vector to unit length. Does your answer make sense?